Agriculture Modernization, Investment, and Structural Change*

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Abstract

This paper illuminates the role played by agricultural modernization in structural change. As workers leave traditional agriculture sector to settle down in modern agriculture sector and non-agriculture sectors, demand for capital goods temporally increases. Since the majority of capital goods comes from the manufacturing sector, the manufacturing employment share would first rise, then decline and converge to a generalized balanced growth path defined by Kongsamut, Rebelo, and Xie (2001). Our model generates hump-shaped patterns for manufacturing employment share and investment rate without assuming unbalanced technology growth.

JEL code: O13 O14 O41

Keywords: agricultural modernization, structural transformation, stages of economic development, hump-shaped pattern

*We thank to Ngo Van Long, Markus Poschke, and Francisco Alvarez-Cuadrado for their invaluable comments and suggestions. Early versions of this paper were presented at the 17th World Congress of the International Economic Association, Jordan, and seminar at Capital University of Economics and Business. We wourld like to acknowledge the financial support from the Research and Innovation Centre of Metropolis Economic and Social Development at Capital University of Economics and Business. And Zongye Huang thanks the financial support from the Humanities and Social Science Research Project of the Ministry of Education of the People's Republic of China (Project No. 16YJC790035).

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1 Introduction

One of the strongest relationships established in the literature of structural change is that the manufacturing employment share exhibits a hump-shaped pattern during economic growth. Two broad sets of explanations have been proposed. The first explanation refers to unbalanced technology growth, which requires technological improvements across industries to satisfy certain patterns (Baumol, 1967; Ngai and Pissarides, 2007).

A second set of explanations emphasizes the role of international trade on structural change (Echevarria, 1995; Uy, Yi, and Zhang, 2013; Huang, 2015). Within limits, foreign trade can be an important factor in determining industry structures. As Uy, Yi, and Zhang (2013) showed, in countries which produce manufactured products for the rest of the world, the manufacturing sector expands to meet foreign demand. But even in such exporting countries, the service sector eventually takes over and the manufacturing employment share shrinks, generating a hump-shaped pattern. Huang (2015) estimates that trade deficits in the United States is associated with a large portion of the decline in the manufacturing sector.

Although these two explanations are insightful, evidence from several sources suggests that a complementary explanation of structural transformation exists. Kuznets (1966) pointed out that structural change is one of the most prominent features of development. And the experiences of various countries are surprisingly similar with each other (Bah, 2011; Herrendorf, Rogerson, and Valentinyi, 2014). Therefore, this paper intends to provide a new channel of structural change that universally applies to almost all countries.

In searching for additional explanations for the rise and fall of the manufacturing sector, it is natural to look for systematic changes during long-term economic growth. Here two factors are of general importance: (1) the agriculture sectors in developed and developing countries are using very different technologies (Priyo, 2012; Donovan, 2016), thus, the technology switch in the agriculture sector, or “agriculture modernization”, can be a key driver of modern economic growth (Yang and Zhu, 2013); (2) as per capita income rises, investment rate exhibits another hump-shaped pattern, which directly affects the manufacturing sector, because expenditure on capital goods is skewed towards manufactured goods.[1]

[1] Kongsamut, Rebelo, and Xie (2001) documented that about 90% to 93% of investment

The aim of this paper is to incorporate these two factors into a general explanation of structural change. Our model highlights the importance of agricultural modernization as a central mechanism that triggers the transition from traditional economy to modern growth. When agriculture relies on traditional technology, which is labor-intensive and does not improve overtime, the agriculture sector has to occupy a large portion of the labor force to meet the subsistence level of food consumption. Nevertheless, in this economy, there exists a modern agriculture technology that uses reproducible capital as a key input and its efficiency persistently improves at an exogenous rate. After passing a certain threshold, the modern technology becomes superior to the traditional technology and is gradually adopted for agriculture production. This modernization of agriculture is an endogenous choice by farmers and affects the demand of capital goods (investment) in two ways. First, the adoption of modern technology requires capital inputs, which directly cause investment demand to rise. Second, it releases excess workers to other sectors who have to accumulate capital goods to move and settle down. Both channels increase the demand for physical capital that are mostly produced by the manufacturing sector. As a result, agriculture modernization causes both investment rate and manufacturing employment to increase (industrialization). When the majority of workers in the traditional agriculture sector have moved into other sectors, the agriculture modernization completes, investment demand for manufactured goods peaks and begins to shrink, which causes the manufacturing employment share to drop (de-industrialization).[2] In order to investigate the timing and mechanism of this transition process, we divide economic development into four sequential stages: traditional stage, mixed stage, convergent stage, and generalized balanced growth path (GBGP) stage.

There are a few papers that have discussed the pattern of the investment

goods were produced by the manufacturing sector during the period from 1958 to 1987 in the United States. The World Input-Output Database (Timmer, 2012) shows that about 85% to 92% of investment goods are produced by the manufacturing sector in developing countries, while this ratio for developed countries is about 70% to 85%. For example, in the year 2000, this ratio was 91% for China, India, and Mexico; 86% for Brazil and Turkey; 85% for Taiwan; 82.5% for Japan; 80.5% for the United States; and about 70% for France and Sweden.

[2] An alternative way to meet this increased investment demand is importing capital goods while leaving domestic industrial production largely unaffected. However, except in a few countries that have abundant natural resources to export or can attract large volumes of capital inflows, this trade pattern is unsustainable. In addition, the impact from high investment demand can not be completely offset through trade, because the dominant component of investment is construction which is largely non-tradable.

rate with structural change. Laitner (2000) observed the rise of the saving rate during economic development and argued that this phenomenon can be explained by the decline of the agriculture sector, because only the accumulation of reproducible capital in non-agriculture sector is recognized by the national income account as savings. Echevarria (2000) pointed out that poor countries are constrained by the subsistence demand for food, thus, they cannot save/invest much. As they get richer, their saving rates increase. Both Echevarria (2000) and Laitner (2000) recognized the rise of investment rate at early stage of development, but ignored the fact that the investment rate decreases as per capita income increases further.

In addition, our framework is related to a burgeoning body of literature that stresses the important role of a technology adoption in the agriculture sector. Hansen and Prescott (2002) constructed a two-sector model with one single final good to track the transition out of a stagnant Malthusian economy. Gollin, Parente, and Rogerson (2007) considered three types of agricultural technologies, one traditional technology and two modern technologies, to calibrate the experience of the United Kingdom over the last 200 years, and argued that food constraints can delay industrialization. But they did not emphasize the role of capital accumulation in such a structural transformation process. Yang and Zhu (2013) endogenized the decision of adopting technology in agricultural production by introducing intermediate goods supplied by the non-agricultural sectors. Since their production functions have no physical capital, there is no role for capital accumulation to play.

The rest of this paper is organized as follows. Section 2 documents some stylized facts of structural change, including the difference of agriculture technologies at different income levels, the employment pattern in the manufacturing sector, and the change of investment rate. Section 3 presents the basic structures of the model and characterizes the economic equilibrium. Section 4 presents our theoretical results and section 5 uses a numerical example to demonstrate the dynamic features of this model. Section 6 provides three pieces of empirical evidence to support the mechanism proposed in this paper. And section 7 concludes.

2 Facts and Motivating Evidence

We document the following three facts. While none of them is absolutely new, our contribution is to provide a unified explanation for them.

Fact 1. *The structure of the agriculture sector in developing countries is significantly different from that in developed countries.*

The agriculture sector is often the largest and dominant sector in developing countries,[3] and its structure is quite different from that in high-income countries. First, as Caselli (2005) and Restuccia, Yang, and Zhu (2008) explained, differences in labor productivity in the agricultural sector are much larger than differences in non-agricultural labor productivity. Second, the cross-country difference of capital intensity is also larger in agriculture. Capital per worker in non-agriculture sectors in the richest 10% of the countries is 6.6 times of that in the poorest 10%, whereas the ratio for the agricultural sector jumps 30 times to 204.5 between the richest 10% and their poorest counterparts (Priyo, 2012). Third, farmers in developing countries use fewer intermediate inputs. As a share of harvest value, it ranges from 4 percent in Uganda to 40 percent in the United States (Donovan, 2016).

Based on these observations, it is evident that the agriculture sector in poor countries possesses a very small capital stock and use very little intermediate inputs, but employ a large portion of the labor force. We argue that we should separate this sector from other sectors, including the modern farming industry, and consider it as a traditional subsistence sector.

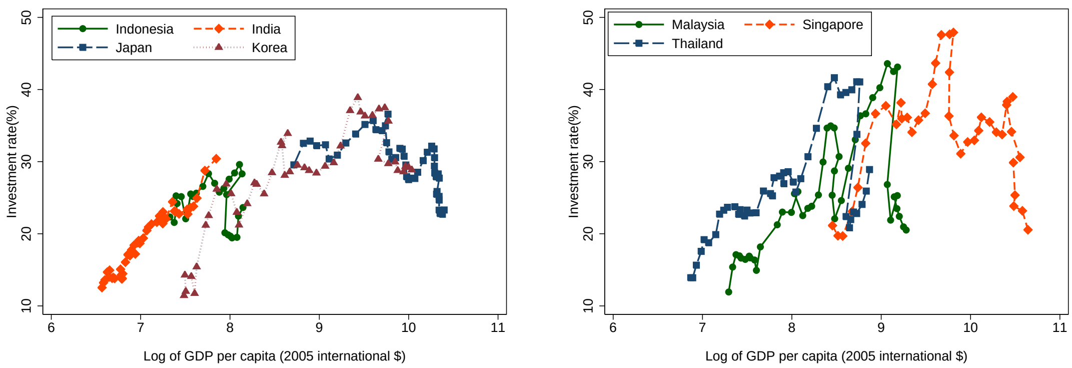
Fact 2. *The employment share in the manufacturing sector exhibits a hump-shaped pattern during economic development.*

The hump-shaped pattern of manufacturing employment has been well documented in the literature. Kuznets (1966, 1971, 1973) discussed the industrialization process in which labor leaves the agriculture sector and enters industry and services sectors. Rowthorn and Ramaswamy (1999) and Rowthorn and Coutts (2004) reviewed de-industrialization in the OECD countries. Bah (2011) discussed the paths of structural change for different groups of countries. And Herrendorf, Rogerson, and Valentinyi (2014) documented the trend of employment shares using several different data sources.

[3] In 2000, the World Bank reported that in economies with a per capita GDP less than one thousand dollars, about 49% of the employment share is held by the agricultural sector.

Fact 3. *During the process of development, the investment rate first increases with income and then gradually decreases, following a hump-shaped pattern.*

The pattern of investment rates is not as obvious as the pattern of employment shares, since investment is more volatile. The positive correlation between investment rate and per capita income is well-known in the literature of economic growth, however, the decrease of investment rate at relatively high income level has been insufficiently emphasized.



(a) Indonesia, India, Japan, and Korea (b) Malaysia, Singapore, and Thailand
Source: World Development Index and author's calculations.

Figure 1: Investment rates in selected Asian countries

We start by investigating the long-term trend of investment rate in a set of countries. Kuznets (1966, Table 5.5) documented the investment rate over a period of approximately a century (1860-1960 in most cases), and the World Development Index provides the investment shares measured by fixed capital formation since 1960. The ratio of net investment to output evolved in Australia from 12% to 30% before hitting 22%; it evolved in Denmark and Italy from 5% to more than 25% in the 1960s before dropping to 20% in the 2000s, and in Canada from 7% to 25% and down to 20%; in Japan and Korea, it evolved from 6% to 36% and then 23%, and from 6% to 40% and down to 30%, respectively.

Figures 1 panel (a) and (b) display the patterns of investment rate for seven Asian countries: Indonesia, India, Japan, Korea, Malaysia, Singapore, and Thailand. The investment rate firstly increased as income growth peaked at a mid-income level, and decreased thereafter. This hump-shaped pattern is more distinct in countries that have been able to achieve sustained growth, such as Korea, Japan, Malaysia, Singapore, and Thailand.

Finally, we run the following regression to test the hump-shaped relationship between investment rate and per capita income:

$$Inv_{i,t} = \beta_0 + \beta_1 \log(y_{i,t}) + \beta_2 \left[\log(y_{i,t})\right]^2 + \beta_3 Z_{i,t} + \epsilon_t,$$

where $Inv_{i,t}$ is the investment rate (%) for country i in time period t, $\log(y_{i,t})$ is PPP GDP per capita in constant 2005 international dollars. We also include variable $Z_{i,t}$ in the regression to control for country-specific factors and time trend. The results are given in Table 1. The estimated coefficients β_1 and β_2 are statistically significant at the 1 percent level, and β_2 is negative, suggesting a hump-shaped pattern.

Table 1: Pattern of investment rate

	Dependent variable: Investment rate		
	(1)	(2)	(3)
Income per capita	14.718***	32.118***	29.237***
	(0.594)	(1.369)	(1.373)
Income per capita squared	-0.8739***	-1.881***	-1.708***
	(0.0374)	(0.085)	(0.090)
Country dummies	N	Y	Y
Time dummies	N	N	Y
No. of obs.	6812	6812	6812
R^2	0.094	0.479	0.502

Notes: All regressions are OLS. Standard errors are in parentheses. Robust standard errors are clustered at country level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

3 The Model

We construct a four-sector model with three types of goods: food, manufactured product, and services. The four sectors are traditional agriculture, modern agriculture, manufacturing, and services, indexed by subscript 0 to 3. In order to maintain a structure that is as close as possible to standard growth models with structural change,[4] we abstract from the presence of land and the

[4]For example, Kongsamut, Rebelo, and Xie (2001) used a similar model without the modernization of the agriculture sector.

presence of international trade.[5]

3.1 Economic Environment

Preference

The economy is populated by an infinitely lived representative family. For the sake of simplicity, we normalize family size to one. Since each member of the household provides one unit of labor to the market inelastically every period, the aggregate labor supply is one unit. Therefore, the labor movements across sectors are equivalent to the time allocation of the representative agent.

We assume that preferences are time separable and include different income elasticities in our utility specification:

$$U(C_{i,t}) = \begin{cases} 0, & \text{if } C_{1,t} < \bar{C}_1, \\ \gamma \log C_{2,t} + (1-\gamma)\log(C_{3,t} + \bar{C}_3), & \text{if } C_{1,t} = \bar{C}_1, \end{cases} \tag{1}$$

where $\bar{C}_1 \geqslant 0$ is a subsistence level of food consumption that embeds our version of Engel's law. This utility function is very similar to the one used in Kongsamut, Rebelo, and Xie (2001), except that we impose a more restrictive condition on agriculture consumption that is constrained by an upper bound, $\bar{C}_1$.[6] Households with a low living standard struggle to survive ($C_{1,t} < \bar{C}_1$), while households with a high standard of living would become satiated with $C_{1,t} = \bar{C}_1$ and devote their remaining expenditures exclusively to goods from other sectors. $\bar{C}_3 \geqslant 0$ can be viewed as representing home-produced services.

Lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t U(C_{i,t}), \tag{2}$$

where β is the subjective time discount factor.

[5] Food imports and food aid are not a major source of food in poor countries. Using the United Nations Food and Agriculture Organization (FAO) data, Gollin, Parente, and Rogerson (2007) claimed that net imports of food supplied around 5% of total calorie consumption in 2000 for all low-income countries. They concluded that it is reasonable to view most economies as closed from the perspective of trade in food. Therefore, most of the resources in agriculture are used domestically to meet domestic food needs.

[6] Laitner (2000) and Gollin, Parente, and Rogerson (2007) made similar assumptions in their two-sector models.

The budget constraint of the family is given by

$$\sum_{i\in\{0,1,3\}} P_{i,t}Y_{i,t} + P_{2,t}(C_{2,t} + I_t) = w_t + r_tK_t. \tag{3}$$

Technology

There are two types of technologies in the economy. Both traditional and modern technologies are potentially available to produce food, while only modern technology is used in producing non-agriculture goods and services.

Traditional technology uses labor as the only input,[7]:

$$Y_{0,t} = B_{0,t}N_{0,t}. \tag{4}$$

while modern technology uses both labor and capital, which is similar across sectors:

$$Y_{i,t} = B_{i,t}\left(\phi_{i,t}K_t\right)^{\alpha}\left(X_tN_{i,t}\right)^{1-\alpha}, \quad i = 1, 2, 3, \tag{5}$$

where $B_{i,t}$ and $i \in \{0, 1, 2, 3\}$ are relative productivity indexes. For the sake of simplicity, we assume $B_{i,t}$ to be constant, $B_{i,t} = B_i$, and set $B_{2,t} \equiv 1$. $\phi_{i,t}$ represents the capital allocation for sector i at time t. And $N_{i,t}$ are labor inputs. One thing worth noting is that the labor employment of the agriculture sector is the sum of both traditional and modern agriculture sectors, $N_{A,t} = N_{0,t} + N_{1,t}$. Finally, variable X_t denotes the level of technological progress, which is assumed to be labor augmenting and to increase at an exogenous rate g,

$$\frac{X_{t+1}}{X_t} = 1 + g, \text{ and } X_t > 0,\ g > 0. \tag{6}$$

As shown in the household preference, if $\frac{\bar{C}_1}{B_0} \geqslant 1$, only the traditional agriculture sector exists. This economy struggles to survive. However, a more interesting scenario would include all three types of consumption goods, thus we make the following assumption.

Assumption 1. $\frac{\bar{C}_1}{B_0} < 1$.

Since the production functions exhibit constant returns to scale, we assume there is one competitive firm operating in each sector. Given a wage rate (w_t)

[7]Land is a key input in farming. However, since we have ignored population growth, quantity of arable land would be constant all the time.

and a capital rental rate (r_t), the firm in sector i and $i \in \{0,1,2,3\}$ solves the following problem:

$$\max \{P_i Y_i - w_t N_{i,t} - r_t \phi_{i,t} K_t\}, \tag{7}$$

subject to the production functions above. Given δ as the depreciation rate, the capital accumulation is usual,

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{8}$$

where all capital goods are produced by the manufacturing sector.

3.2 Market Equilibrium

Definition 1. A *competitive equilibrium* is a sequence of relative final good prices $\{p_{i,t}\}_{t\geqslant 0}$, factor prices $\{w_t,\ r_t\}_{t\geqslant 0}$, household consumption bundles $\{C_t, C_{i,t}\}_{t\geqslant 0}$, labor allocations $\{N_{i,t}\}_{t\geqslant 0}$, capital allocations $\{\phi_{i,t}\}_{t\geqslant 0}$, and capital stock $\{K_t\}_{t\geqslant 0}$, such that the followings are true:

(i) Given the sequence of prices, the household maximizes equation (2) subject to budget constraint, equation (3);

(ii) Given the sequence of prices, firms employ labor and capital to solve the allocation problem specified in equation (7);

(iii) All markets clear:

$$\begin{aligned} Y_{0,t} + Y_{1,t} &= \bar{C}_1, \\ Y_{2,t} - I_t &= C_{2,t}, \\ Y_{3,t} &= C_{3,t}, \\ \sum_{i\in\{0,1,2,3\}} N_{i,t} &= 1, \\ \sum_{i\in\{1,2,3\}} \phi_{i,t} &= 1. \end{aligned}$$

If we let $y_{i,t} = \frac{Y_{i,t}}{X_t N_{i,t}}$ and $k_{i,t} = \frac{\phi_{i,t}}{N_{i,t}}\frac{K_t}{X_t}$, equation (5) can be rewritten as

$$y_{i,t} = B_{i,t} k_{i,t}^{\alpha}, \quad i = 1,2,3. \tag{9}$$

Since capital and labor are freely mobile, an efficient allocation requires that the marginal rate of transformation be equated across all production sectors, which implies

$$\frac{\phi_{i,t}}{N_{i,t}} = \frac{1}{1 - N_{0,t}}, \tag{10}$$

$$k_{i,t} = \frac{K_t}{(1 - N_{0,t})X_t} \equiv k_t, \tag{11}$$

where k_t is the amount of capital per unit of effective labor across sectors.

The relative prices, $p_{i,t}$, are determined by the relative productivity and capital income shares, such as

$$\begin{aligned} p_{0,t} &= \frac{P_{0,t}}{P_{2,t}} = \frac{B_2}{B_0} (1-\alpha) k_t^{\alpha} X_t, \\ p_i &= \frac{P_{i,t}}{P_{2,t}} = \frac{B_2}{B_i}, \quad i \in 1, 2, 3. \end{aligned} \tag{12}$$

Using these relative prices, the total output of the modern sectors satisfies

$$C_{2,t} + I_t + p_1 Y_{1,t} + p_3 Y_{3,t} = B_2 (K_t)^{\alpha} \left[(1 - N_{0,t}) X_t\right]^{1-\alpha}. \tag{13}$$

The competitive equilibrium for this economy characterizes the optimal allocation of consumption across sectors

$$\frac{p_3(C_{3,t} + \bar{C}_3)}{C_{2,t}} = \frac{1-\gamma}{\gamma}, \tag{14}$$

and the Euler equation is given by

$$\frac{C_{2,t+1}}{C_{2,t}} = \beta(r_{t+1} + 1 - \delta), \tag{15}$$

where the capital rental rate satisfies

$$r_{t+1} = \alpha B_2 k_{t+1}^{\alpha-1}. \tag{16}$$

Based on the fact that most capital goods come from the manufacturing sector, we assume that all capital goods are produced by the modern manufacturing sector. Using the production function and equilibrium conditions, the labor input in the manufacturing sector is given by

$$\begin{aligned} N_{2,t} &= \frac{C_{2,t}+I_t}{B_2 k_t^{\alpha} X_t}, \\ &= \gamma(1-N_{0,t}) + \frac{\gamma}{k_t^{\alpha} X_t}\left(\frac{B_0}{B_1}N_{0,t} + \frac{\bar{C}_3}{B_3} - \frac{\bar{C}_1}{B_1}\right) + \frac{(1-\gamma)I_t}{B_2 k_t^{\alpha} X_t}, \end{aligned} \tag{17}$$

where investment, I_t, satisfies

$$I_t = \left[(1+g)(1-N_{0,t+1})k_{t+1} - (1-\delta)(1-N_{0,t})k_t\right] X_t. \tag{18}$$

The investment rate, denoted by iy_t, is defined as

$$iy_t \equiv \frac{I_t}{Y_t}. \tag{19}$$

Equation (17) indicates that the employment in the manufacturing sector is not only affected by investment (the third component), but is also closely related to the consumption demand (the first two components) which can be directly affected by the traditional agriculture sector ($N_{0,t}$). Therefore, the linkage between the manufacturing employment and investment is not trivial.

4 Four Stages of Economic Growth

The following proposition summarizes the conditions of technology adoption in the agriculture sector.

Proposition 1. *If we let* $Z_t = \left(\frac{B_0}{(1-\alpha)B_1}\right)^{\frac{1}{\alpha}} X_t^{-\frac{1}{\alpha}}$, *in agricultural production, the firm*

1) uses only traditional technology, if $k_t < Z_t$;
2) uses only modern technology, if $k_t > Z_t$;
3) and uses a mixed combination, if $k_t = Z_t$.

We assume $X_0 > 0$ is small enough at the beginning of the analysis to ensure that the traditional economy is feasible.[8]

[8] To ensure production in the service sector, X_0 has to satisfy

$$X_0 > \frac{B_1 B_2}{B_3^2}\frac{\gamma}{1-\gamma}\frac{\bar{C}_1}{\left[B_2 k_0^{\alpha} - k_0\,(g+\delta)\right](1-\frac{\bar{C}_1}{B_0})}.$$

Proposition 1 suggests that long-term economic growth can be divided into multiple stages. In the following definition, we consider economic development as a four-stage process.

Definition 2. Four stages of economy growth.

Traditional Economy: agriculture production only uses traditional technology.

Mixed Economy: both traditional and modern technologies are equally efficient, agriculture production starts to adopt modern technology.

Convergent Economy: only modern technology exists in agricultural production, the economy converges to a generalized balanced growth path through a capital accumulation process.

GBGP Economy: the economy evolves along a generalized balanced growth path.

In the rest of this section, these four stages will be discussed in turn.

4.1 GBGP Economy

We start with the last stage, the fourth stage, where this economy performs in a way that is close to the standard growth model. The moment that the economy reaches its generalized balanced growth path is denoted by G. We will show that under certain assumptions, this economy can grow along the generalized balanced growth path (GBGP), which is defined by Kongsamut, Rebelo, and Xie (2001) as follows.

Definition 3. A *generalized balanced growth path* is a trajectory with a constant real interest rate.

In addition, we make the following assumption to ensure the existence of a generalized balanced growth path.[9]

Assumption 2. $\frac{\bar{C}_1}{\bar{C}_3} = \frac{B_1}{B_3}$.

Proposition 2. *Whenever assumption 2 holds, $\frac{\bar{C}_1}{\bar{C}_3} = \frac{B_1}{B_3}$, a generalized balanced growth path exists. Relative prices, aggregate labor income share, and the growth rate of output and capital are constant. The employment share declines in agriculture, rises in services, and remains stable in manufacturing.*

[9] See Kongsamut, Rebelo, and Xie (2001) for more details.

The capital rental rate and sectoral capital per effective labor consistent with this generalized balanced growth path are given by

$$\bar{r} = \frac{1+g}{\beta} + \delta - 1. \tag{20}$$

$$\bar{k} = \left(\frac{\alpha B_2}{\bar{r}}\right)^{\frac{1}{1-\alpha}} \tag{21}$$

The total output $Y_t = B_2\bar{k}^\alpha X_t$, and the amount of capital goods used for investment satisfies $I_t = (g+\delta)\bar{k}X_t$, thus, the investment rate is also constant,

$$iy_t = \frac{g+\delta}{B_2}\bar{k}^{\alpha-1} \equiv iy_G. \tag{22}$$

Since the total demand of agriculture product is given by $\bar{C}_1$, the agriculture employment is

$$N_{1,t} = \frac{\bar{C}_1}{B_1 k_t^\alpha X_t}, \tag{23}$$

at time G, $N_{1,G} = \frac{\bar{C}_1}{B_1\bar{k}^\alpha X_G}$.

Similar to the result of Kongsamut, Rebelo, and Xie (2001), the dynamic reallocation of labor across sectors is given by

$$\Delta N_{1,t} = N_{1,t+1} - N_{1,t} = -\frac{g}{1+g}\frac{\bar{C}_1}{B_1\bar{k}^\alpha X_t}, \tag{24}$$

$$\Delta N_{2,t} = N_{2,t+1} - N_{2,t} = 0, \tag{25}$$

$$\Delta N_{3,t} = N_{3,t+1} - N_{3,t} = \frac{g}{1+g}\frac{\bar{C}_3}{B_3\bar{k}^\alpha X_t}. \tag{26}$$

Since $N_{2,t}$ is constant, $N_{2,t} = N_{2,G} = \gamma + \frac{1-\gamma}{B_2}(g+\delta)\bar{k}$. These equations show that along the generalized balanced growth path, the share of labor in agriculture continues to decline, the share in manufacturing remains constant, and the share in services expands. In the long run, these rates converge to zero, as X_t continues to grow.

4.2 Traditional Economy

When only traditional technology is operated in the agriculture sector, it is a traditional economy. According to proposition 1, $k_t < Z_t$ should be satisfied to ensure that the traditional production technology is more cost-efficient than the modern technology in food production.

As we assume the traditional agriculture sector only uses labor as input, while the modern technology requires both capital and labor, the whole economy in this traditional stage can be divided into to two systems: a traditional system employs a large portion of labor, and a modern system that uses reproductive capital goods.

The share of workers employed in the agriculture sector remains constant through the traditional stage, such that

$$\bar{N}_0 = \frac{\bar{C}_1}{B_0}. \tag{27}$$

Since no firms use modern agriculture technology to produce outputs, the rest of labor, $1 - \bar{N}_0$, is employed by the manufacturing sector and the service sector.

The following descriptions summarize key characteristics of this traditional economy. The agriculture sector employs a large portion of the labor force and uses the traditional technology to meet the subsistence food consumption. The value-added share of output of this traditional agriculture sector is less than the employment share.[10] There is no sign of industrialization that encourages workers to leave the agriculture sector and join the modern sectors, because the traditional technology is more efficient.

4.3 Mixed Economy

Between the traditional economy and the GBGP economy, there are two interesting stages that contain complex dynamic features. We start with the mixed economy, where a combination of traditional and modern agriculture technologies is used.

Starting with an economy in the traditional stage, as the exogenous technological progress, X_t, continues to improve, the relative price of output produced by the traditional agriculture technology becomes more and more expensive. The equality condition in proposition 1 will eventually holds and farmers start

[10] As we assume $\frac{\bar{C}_1}{B_0} < 1$, the other two sectors, manufacturing and services, will employ $1 - \bar{N}_0$. Given wage rate w_t, the value added per worker in the traditional agriculture sector is $v_{0,t} = \frac{p_{0,t}\bar{C}_1}{\bar{N}_0} = B_2 (1-\alpha) k_t^\alpha X_t$, while the value added per worker in the modern sectors is $v_{2,t} = B_2 k_t^\alpha X_t$, thus, $v_{0,t} < v_{2,t}$.

to adopt the modern technology for agricultural production. At that point, marked by subscript M, since both technologies are operated simultaneously, the economy enters the mixed stage. We have $Z_M = \left(\frac{B_0}{(1-\alpha)X_M B_1}\right)^{\frac{1}{\alpha}} = k_M$. Thus, continued industrial productivity growth, or a persistent increase in the relative price of traditional agriculture products, eventually triggers the transition from traditional agriculture production to modern agriculture production.

Throughout the duration of the mixed economy,[11] the equality condition in proposition 1 has to be maintained, which determines the evolution of the capital-per-effective-labor ratio, $k_t = \left[\frac{B_0}{B_1(1-\alpha)X_t}\right]^{\frac{1}{\alpha}}$. Thus, $k_t < k_{t-1}$ as $X_t > X_{t-1}$.

Proposition 3. *In the stage of mixed economy, for any time $t > M$, if $N_{0,t} > 0$, $N_{1,t} > 0$, thus*

$$k_t = \left[\frac{B_0}{B_1(1-\alpha)X_t}\right]^{\frac{1}{\alpha}}, \tag{28}$$

where k_t satisfies $k_t = \frac{k_{t-1}}{(1+g)^{1/\alpha}} < k_{t-1} \leqslant k_M$.

Proposition 3 suggests that because the employment adjustment in the traditional agriculture sector is not instant, k_t gradually decreases.

Using the optimal consumption allocation, equation (14), and assumption 2, the aggregate consumption is a function of $C_{2,t}$, such as

$$\begin{aligned} p_1\bar{C}_1 + C_{2,t} + p_3C_{3,t} &= C_{2,t} + p_3(C_{3,t} + \bar{C}_3) \\ &= \frac{C_{2,t}}{\gamma}. \end{aligned} \tag{29}$$

Proposition 4. *The total output of this economy in the mixed stage is determined by the process of agriculture modernization,*

$$Y_t = \frac{B_2B_0}{B_1(1-\alpha)}(1-\alpha N_{0,t}); \tag{30}$$

[11] With certain parameter values, for example, $\bar{N}_t$ is very small or g is large enough, the stage of mixed economy can last less than one period, meaning that equation (28) does not hold even for k_{M+1}. However, this issue is only caused by the modeling approach of discrete time with a fixed time interval. We can change the unit of time interval, or rewrite these equations in continuous time, to deal with this problem. If the equation of $N_{0,t}$ is differentiable at any time t, and $N_{0,M} > 0$, it will take time $t_M > 0$ before $N_{0,t}$ reaches 0, during which equation (28) holds with $\dot{X}_t = g$, and $\dot{k}_t = -\frac{g}{\alpha}$.

The investment rate in the mixed stage satisfies

$$iy_t = \frac{\left[(1-N_{0,t+1})(1+g)^{\frac{\alpha-1}{\alpha}} - (1-\delta)(1-N_{0,t})\right]}{B_2(1-\alpha N_{0,t})}\left(\frac{B_0}{(1-\alpha)B_1X_t}\right)^{\frac{1-\alpha}{\alpha}}. \quad (31)$$

This proposition shows that the total output is only affected by the change of $N_{0,t}$, the labor employment in the traditional agriculture sector, meaning that the agriculture modernization is the primary driving force for economic growth in this mixed stage. Moreover, the change of $N_{0,t}$ is also crucial of determining the investment rate, iy_t. A rapid decline in the traditional agriculture sector leads to a higher investment rate.

The following proposition, proposition 5, summarizes the key dynamic features of employment share movements across sectors.

Proposition 5. *The movements of employment shares in the mixed economy exhibit the following properties:*

1) Total employment shares used to produce agriculture goods start to decline,

$$N_{A,t+1} - N_{A,t-1} = \alpha(N_{0,t+1} - N_{0,t}); \quad (32)$$

2) The size of the traditional agriculture sector, in terms of employment, is given by

$$\frac{G(N_{0,t+1})}{G(N_{0,t})} = \beta(\alpha B_2 k_t^{\alpha-1} + 1 - \delta), \quad (33)$$

where k_t is given by equation (28) and $G(N_{0,t+1}) = C_{2,t}$ satisfies

$$\begin{aligned} G(N_{0,t+1}) \equiv & -\gamma\left[\frac{1-N_{0,t+1}}{(1+g)^{\frac{1-\alpha}{\alpha}}} - (1-\delta)(1-N_{0,t})\right]\left(\frac{B_0}{(1-\alpha)B_1}\right)^{\frac{1}{\alpha}} X_t^{\frac{\alpha-1}{\alpha}} \\ & + \frac{\gamma B_2 B_0}{B_1}\frac{1-\alpha N_{0,t}}{1-\alpha}; \end{aligned} \quad (34)$$

3) The employment shares in the manufacturing and service sectors are given by

$$N_{2,t} = \gamma(1-\alpha N_{0,t}) + \frac{1-\gamma}{B_2}\left[\frac{1-N_{0,t+1}}{(1+g)^{\frac{1-\alpha}{\alpha}}} - (1-\delta)(1-N_{0,t})\right]\left(\frac{B_0}{(1-\alpha)X_tB_1}\right)^{\frac{1-\alpha}{\alpha}}, \quad (35)$$

$$N_{3,t} = \frac{B_1(1-\alpha)}{B_0B_3}\left[\frac{1-\gamma}{\gamma}\frac{G(N_{0,t+1})}{p_3} - \bar{C}_3\right]. \quad (36)$$

Unfortunately, proposition 5 cannot make clear predictions on the movements of the manufacturing employment share, which is the key variable that we care about most. However, based on the mechanism described in this stage, the switch of agriculture production from traditional technology into modern technology causes a rapid decline of the agriculture employment share that should be shared by the manufacturing and the service sectors. Equation (35) indicates that manufacturing employment has two components on the right-hand side. The first component would increase as $N_{0,t}$ drops to zero. The second component is associated with the demand for investment in this economy. There are two opposite factors, $\left(\frac{B_0}{(1-\alpha)X_t B_1}\right)^{\frac{1-\alpha}{\alpha}}$ decreases as X_t rises, while the direction for $(1 - N_{0,t})(\frac{1}{(1+g)^{\frac{1-\alpha}{\alpha}}} - 1 + \delta) - \frac{\Delta N_{0,t+1}}{(1+g)^{\frac{1-\alpha}{\alpha}}}$ is uncertain. Because the lack of capital in this stage drags down the amount of capital per unit of effective labor to maintain the relative price for agriculture products, we argue that the second component in equation (35) should, in general, increase with $1 - N_{0,t}$.

Finally, we would like to show that the length of this mixed stage is finite, meaning that the technology adoption process will complete and move on to the next stage. The reason for the economy to undergo the mixed stage is that workers who leave the traditional agriculture sector need capital goods to settle down in modern sectors. This increased demand for capital can not be satisfied by the economy immediately. However, if we consider the capital per effective labor as the threshold capital requirement for agriculture modernization and structural change, it continuously decreases in the mixed stage (proposition 3), meaning that it becomes easier to adopt modern technology. As a result, the economy can gradually accumulate capital and complete this mixed stage.

4.4 Convergent Economy

At the end of the mixed stage, the employment share in the traditional sector drops to zero. Traditional technology becomes obsolete and all economic activities take place in modern sectors. We claim that at this very moment, which is marked by subscript C, this economy enters the era of a convergent stage with $N_{0,C} = 0$. According to proposition 3, in the mixed economy stage, the capital per effective labor has been decreasing over time, thus, $k_C < k_M$. Therefore, we assume that the capital per unit of effective labor at time C, k_C

is less than $\bar{k}$.[12]

Using the aggregate consumption, equation (29), we let $c_t = \frac{c_{2,t}}{\gamma} = \frac{C_{2,t}}{\gamma X_t}$.[13] And the dynamic features of our model in this convergent stage are similar to a standard Ramsey-Cass-Koopmans growth model with a saddle path that converges to the steady states with $\dot{c}_t = \dot{k}_t = 0$. The two key equations, therefore, are given by

$$\frac{c_{t+1}}{c_t} = \beta(\alpha B_2 k_{t+1}^{\alpha-1} + 1 - \delta)(1+g), \tag{37}$$

$$k_{t+1} = \frac{B_2 k_t^{\alpha} + (1-\delta)k_t - c_t}{1+g}. \tag{38}$$

The Euler equation implies that total consumption will increase at a rate higher than g, for $k_t < \bar{k}$. And on such a saddle path, both k_t and c_t have to increase. However, as k_t approaches $\bar{k}$, the growth rate of c_t decreases and converges to g. In section 5, we construct a numerical example to illustrate the transition dynamics of our model. Figure 2 demonstrates the path of k_t through the four stages. And Figure 7 illustrates the interaction between k_t and c_t, which provides the dynamic paths for both mixed and convergent stages.

The investment rate in this convergent stage is given by

$$iy_t = \frac{(1+g)\,k_{t+1} - (1-\delta)k_t}{B_2 k_t^{\alpha}}. \tag{39}$$

Because $k_C < \bar{k}$, k_t has to gradually rises towards $\bar{k}$ along a saddle path. This process requests capital accumulation, thus, iy_t has to be higher than iy_G on the GBGP. However, as k_t is getting closer to $\bar{k}$, investment demand decreases and converges to the long-run steady state.

Sectoral employment shares, in this convergent stage, are given by

$$N_{0,t} = 0, \tag{40}$$

[12] This assumption, which appears to be a little arbitrary, is consistent with economic intuition. If we assume $k_C > \bar{k}$, since $k_M > k_C$, k_M is strictly larger than $\bar{k}$, which implies $r_M < \bar{r}$. This result is counter-intuitive, as on average the real interest rate is higher in countries with lower incomes. For example, the World Development Index shows that the average real interest rate decreased as income level increased in 2005.

[13] $c_{2,t} = \frac{C_{2,t}}{(1-N_{0,t})X_t}$, as $k_t = \frac{C_{2,t}}{(1-N_{0,t})X_t}$. Since $N_{0,t} = 1$ in the convergent stage, $c_t = \frac{c_{2,t}}{\gamma} = \frac{C_{2,t}}{\gamma X_t}$.

$$N_{1,t} = \frac{\bar{C}_1}{B_1 k_t^{\alpha} X_t}, \tag{41}$$

$$N_{2,t} = \gamma + \frac{1-\gamma}{B_2} \left[(1+g) \frac{k_{t+1}}{k_t^{\alpha}} - (1-\delta) k_t^{1-\alpha} \right], \tag{42}$$

$$N_{3,t} = 1 - N_{1,t} - N_{2,t}. \tag{43}$$

The equation for $N_{1,t}$ shows that the agriculture employment share decreases as it is on the generalized balanced growth path. The following proposition shows that manufacturing employment share also decreases in the convergent stage.

Proposition 6. *In the convergent stage, the employment share of the manufacturing sector decreases. If k_t converges along the saddle path to $\bar{k}$, the manufacturing employment share also moves towards $N_{2,G}$ on the generalized balanced growth path.*

Comparing with equation (39), equation (42) shows that the contraction in manufacturing employment during the convergent stage is primarily caused by the decline of investment.

4.5 Summary

The main dynamic features of our model on sectoral employment shares and investment rate are summarized in Table 2, where "TA" and "MA" represent "traditional agriculture" and "modern agriculture", and "A", "M", and "S" stand for the three aggregate sectors. We use "+", "−", "Const.", and "0" to indicate if the parameter would increase, decrease, remain constant, or remain at zero. If the trend of the variable is not clearly predictable, we leave it blank.

Table 2 indicates that the agriculture employment share (both traditional and modern agriculture) is constant in the traditional stage and continues to shrink since the mixed stage, while the service sector expands. The size of the manufacturing sector is more difficult to determine. However, based on our analysis and economic intuition, we expect manufacturing employment to firstly increase in the mixed stage, then decrease in the convergent stage, and eventually reach the generalized balanced growth path, exhibiting a hump-shaped pattern over this four-stage process of economic development. The peak moment is expected to be close to the end of the mixed stage, as the size of the traditional agriculture approaches zero.

Table 2: Summary of the key variable movements

Stages	Sectoral employment					k_t	iy_t
	TA	MA	A	M	S		
Traditional	Const.	0	Const.				
Mixed	−	+	−	$+^*$	+	−	$+^\#$
Convergent	0	−	−	−	+	+	−
GBGP	0	−	−	Const.	+	Const.	Const.

* Trend of manufacturing employment: we expect $N_{2,t}$ to increase in the mixed stage and reach its peak right before entering the convergent stage.
# The investment rate first increases in the mixed stage. As most labor has left the traditional agriculture sector, the investment rate begins to decrease.

The last column of Table 2 summarizes the change of investment rate in the model. It is worth pointing out that the connection between manufacturing employment and investment rate is not trivial. The increase of $N_{2,t}$ in the mixed stage is caused by both high investment demand for capital goods and rising consumption demand for manufactured product, whereas the decline of $N_{2,t}$ in the convergent stage is only caused by shrinking investment demand.

5 A Numerical Example

The structural change arises from a combination of multiple forces, where the agriculture modernization is only one of them. In addition, restrictive assumptions have been made to assure the existence of the generalized balanced growth path in the long run. Therefore, our model might not be flexible enough to replicate the actual patterns of structural transformation. However, it would be helpful to use a simulated numerical example to demonstrate that our model is able to generate the key features of economic growth, such as the agriculture modernization, the rise and fall of investment rates, and the reallocation of labor across industries.

Computationally, we exploit the fact that at the limit the economy converges to a generalized balanced growth path. Therefore, the full transition path can be simulated using the following algorithm. First, we can solve for the steady state on the generalized balanced growth path, using preference parameters and the rate of technology growth. Then, we use two initial boundary conditions in the traditional stage: the initial size of the traditional agriculture sector and the initial capital stock per effective labor, k_0, to pin down the final moment of mixed stage for k_C. Finally, we can compute the transitions in the convergent stage backward from GBGP to k_C.

5.1 Parameter Values

We consider a sample economy that is characterized by the parameters in Table 3. Production parameters, B_i, are normalized to 1.[14] The manufacturing consumption share, γ, is set at 0.15, which is consistent with a relatively low expenditure share on manufacturing products in the long run. In several developed countries, the manufacturing consumption shares are already lower than 15%. For example, in the United States, the manufacturing products only consisted of about 14% to 15% of total consumption between 1996 and 2009. In Japan, this ratio was between 12.9% and 14.2% over the same period.[15] The subsistence demand for food, $\bar{C}_1$, is set at 0.5 in order to match a initial agriculture employment share at 50%. $\bar{C}_3$ is also set at 0.5, according to assumption 2. The household discount factor, β, is set at 0.965, which is a typical value within the range of 0.96 to 0.98 that has been commonly used in the literature.[16] The capital income share is set at 0.5, which is higher than the estimate of Gollin (2002).[17] However, a relatively high capital intensity helps to explain the size of the decline in the agriculture sector. The capital depreciation rate is set to be 0.06, which is consistent with the estimate of McQuinn and Whelan (2007) on the U.S. economy. The last component is the exogenous technology progress, which is set to grow at 0.01.

5.2 Numerical Results

Figure 2 summarizes the change of capital intensities and capital stocks through the four stages. Panel (a) represents the amount of capital stock per unit of efficient labor in the modern sectors, $k_t = \frac{K_t}{(1-N_{0,t})X_t}$, which is the key variable in this model. The numerical simulation result confirms our theoretic pre-

[14]These parameters only determine the relative price across sectors.

[15]These numbers are calculated by the author using the World Input-Output database (Timmer, 2012).

[16]Echevarria (1997) used 0.9743 as the discount factor, and Gollin, Parente, and Rogerson (2007) used 0.96 in their calibration exercise.

[17]Gollin (2002) estimated that the labor income shares across countries are within a range of 0.65 to 0.8.

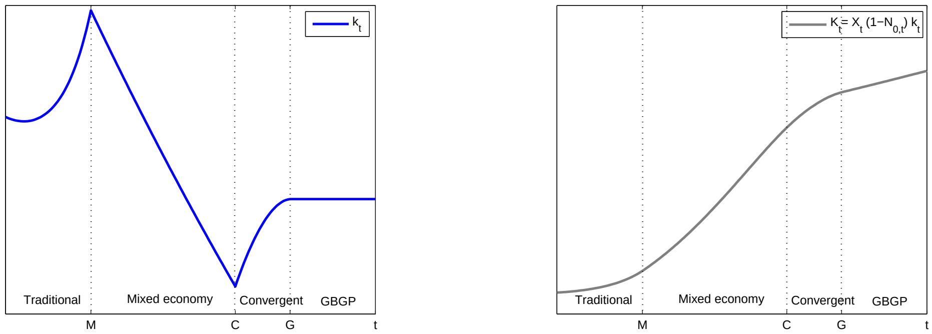
Table 3: Calibration parameters

Parameter		Value	Comments/observations
Preference parameters			
γ	Manufacturing consumption	.15	
$\bar{C}_1$	Subsistence term	.50	Initial agriculture employment at 50%
$\bar{C}_3$	Home service production	.50	Assumption 2
β	Discount factor	.965	Real interest rate around 5%
Traditional agricultural sector			
B_0	Relative productivity	1	Normalization
$\bar{N}_0$	Initial employment share	.50	Initial agriculture employment at 50%
Modern sector parameters			
α	Capital income share	.50	
g	Exogenous technology growth	.01	
δ	Depreciation rate	.06	Estimate of depreciation rate as in McQuinn and Whelan (2007)
B_1	Relative productivity	1	Normalization
B_2	Relative productivity	1	Normalization
B_3	Relative productivity	1	Normalization

dictions. It shows that if the economy is expecting such an industrialization process, it would start to accumulate capital before it enters the mixed economy. Then, the capital per unit of effective labor will fall as labor leaves the traditional sector for modern sectors. As the transition completes, this capital stock per effective labor rebounds and converges to its generalized balanced growth path.

One possible misleading interpretation of the result in Figure 2 panel (a) is that this decline of capital per effective labor in the mixed stage means that the capital stock decreases correspondingly. In panel (b), we show that the capital stock per capita has been rising continuously during the whole process, and its growth rate is particularly high during the mixed stage. This result highlights the primary mechanism of our model that the economy can not provide sufficient amount of capital goods to modernize the traditional agriculture sector, thus capital has to be reallocated to modern agriculture sector and the capital per effective labor ratios decrease, whereas the capital stock per capita continue to increase.

Along the capital-effective-labor ratio path, we can derive the real interest



(a) Sectoral capital per effective labor, k_t (b) Capital stock per capita, K_t

Figure 2: Evolution of capital

rate (capital rental rate) using equation (16), which determines the intertemporal consumption decision, according to the Euler equation of consumption. Then, we can calculate the investment rate, as shown in Figure 3, which shows a hump-shaped pattern. This result is consistent with our empirical observations summarized in section 2. One thing worth noting is that the peak of investment rate takes place in the mixed stage as predicted by Table 2.

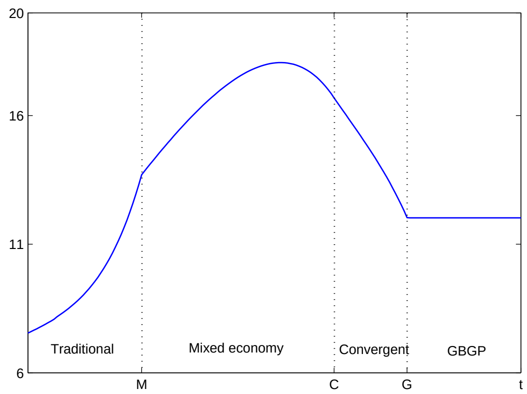


Figure 3: Investment rate (%)

Figure 4 reports the growth rate. In the traditional stage, as the forthcoming industrialization is anticipated, the growth rate increases. As the economy starts to modernize its agriculture production, the growth rate gradually increases and peaks during the mixed stage. For the rest of the mixed stage and the full convergent stage, this output growth rate continues to fall and converges to its long-run level along the generalized growth path. In general, the output growth rate also follows a hump-shaped pattern. The growth rate

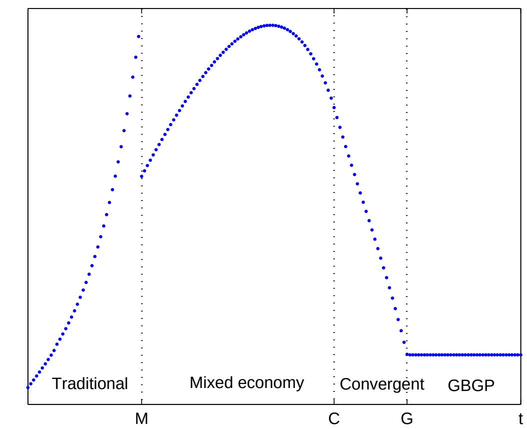


Figure 4: Output growth

tends to be higher during the transition period. This pattern of economic growth rate is qualitatively consistent with empirical evidence. For example, according to Table 9 in Ros, 2001, over the period of 1965 to 1997, the economic growth rate of low-income countries was between 2.6% and 3.1%, that of high-income countries was about 3.4% to 3.5%, while that of middle-income countries was about 4.5%.[18]

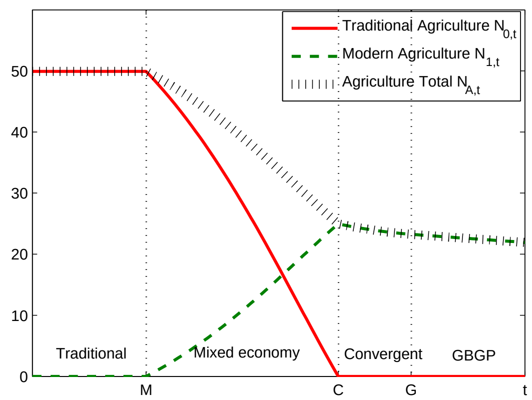


Figure 5: Agriculture employment shares (%)

Next, we turn to look at the agriculture employment shares of the traditional sector, the modern sector, and the aggregate share. As illustrated in

[18]Christiano (1989) and Easterly (1994) obtained a similar hump shape for the growth rate over time series. Easterly (1994) and Echevarria (1997) found a hump-shaped relationship between growth rates and initial income.

Figure 5, in the traditional agriculture stage, because of the subsistence food demand, traditional agriculture employment is stagnant and occupies a significant portion of the labor force. As the economy enters the mixed stage, traditional technology is gradually replaced by modern technology, meanwhile, the aggregate employment of the agriculture starts to decline. At the end of the mixed stage, only the modern agriculture continues to operate and its employment share will decrease further by following the exogenous technology progress.

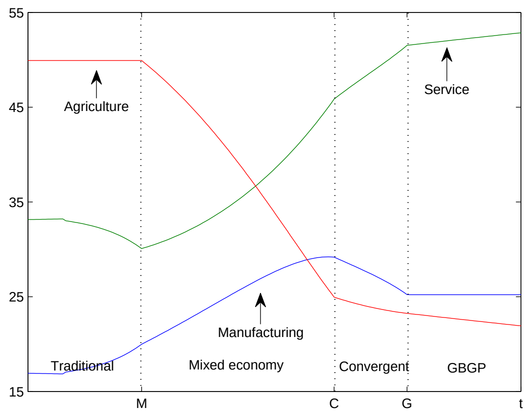


Figure 6: Employment shares of the three sectors (%)

Finally, Figure 6 summarizes the movements of sectoral employment. In the traditional stage, $0 \leqslant t \leqslant M$, the manufacturing sector starts to expand since households anticipate rapid industrialization in the near future, while the agricultural employment occupies a large share (50% of total employment) and remains unchanged. After moment M, the economy enters the mixed stage, agriculture modernization starts, and the employment share of the agriculture decreases rapidly. Meanwhile, the sizes of both the service sector and the manufacturing sector enlarge at similar rates. At moment C, traditional agriculture production becomes obsolete. The employment share of the manufacturing sector peaks and starts to decline and converge to steady-state level. In the long run, the economy will evolve along the generalized balanced growth path as proposed by Kongsamut, Rebelo, and Xie (2001), where the employment share declines in agriculture, rises in services, and is stable in manufacturing.

Therefore, during the mixed stage and the convergent stage, manufacturing employment exhibits a hump-shaped pattern, which fits the key feature of structural change that has been well documented by empirical studies. Comparing manufacturing employment shares with the movements of investment rates (Figure 3), both variables exhibit a hump-shaped pattern, but the investment rate peaks early. This phenomenon reveals the critical difference between these two variables. Investment only captures the dynamic transition in capital stock, while the output from the manufacturing sector can be used for both consumption and investment. Although the investment demand starts to fall in the mixed stage as the agriculture modernization is about to complete, the manufacturing employment share can continue to grow, because the consumption demand for manufactured product is still rising. Therefore, the manufacturing employment share can peak later than investment rate.

5.3 Dynamic Path of k_t and c_t

Phase diagram provides a very convenient way to analyze the transition paths in a standard Ramsey-Cass-Koopmans growth model. Using equations (37) and (38), Figure 7 reports the dynamic path for capital per effective labor, k_t, and consumption per effective labor, c_t.

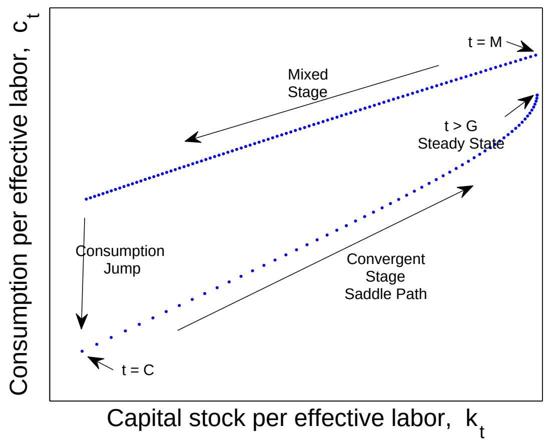


Figure 7: Dynamic path for k_t and c_t

Starting from $t = M$, both variables decrease in the mixed stage, then, at $t = C - 1$, consumption jumps down to a saddle path before entering the convergent stage, and finally, they move along such a saddle path towards the steady state.

6 Empirical Evidence

In this section, we evaluate the following three implications to support our model.

First, it would be interesting to test the primary mechanism proposed in this paper: the agriculture modernization influences structural transformation by affecting demand for capital goods (investment).

Second, our model shows that the peak moment of the manufacturing employment is an important indicator as it is associated with the adoption of modern agriculture technology. Thus, it would be helpful to identify them in the data.

Third, based on our theoretical implications (equations (24) and (41), and proposition 4), we can test the linkage between agriculture modernization and economic growth.

6.1 From Agriculture Modernization to Structural Change

The theoretical analysis and the numerical example have illustrated how agriculture modernization affects structural change, investment, and economic growth: as workers move from traditional agriculture sector to modern agriculture sector, the demand for capital goods raises investment rate, and causes the manufacturing employment to temporarily expand. As the technology adoption completes, the shrinking investment demand causes manufacturing employment share to drop. Therefore, our theory can be schematically summarized as

$$\textit{Agriculture modernization} \overset{①}{\Rightarrow} \textit{Investment rate} \overset{②}{\Rightarrow} \textit{Manufacturing employment}$$

Our first hypothesis implies that the agriculture modernization demands capital goods, affecting investment rates (proposition 4). The basic regression model using panel data is specified as follows,

$$Inv_{i,t} = a_0 + a_1 \cdot \Delta A_{i,t} + a_2 \cdot Z_{i,t} + \epsilon_t, \tag{44}$$

where i indexes countries, $Inv_{i,t}$ is the investment share of GDP for country i at time t, $\Delta A_{i,t}$ measures the intensity of agriculture modernization, And $Z_{i,t}$ refers to a set of control variables that are included in the regression as potential explanations, such as the manufacturing employment share, the output growth rate, the real GDP per capita, country dummies, and time dummies.

This first regression tests whether agriculture modernization affects investment rate, after controlling these relevant explanatory variables. Although the agriculture modernization is not directly observable (the change of $N_{0,t}$ in the model), it can be captured by the change of employment share in the agriculture sector, $\Delta A_{i,t} = N_{A,i,t} - N_{A,i,t-1}$.[19] Therefore, the coefficient of interest is a_1, which reflects whether agriculture modernization influences investment rate. A negative and significant a_1 is interpreted as evidence in favor of the view that countries with rapid structural transition in agriculture production have strong demand for capital goods.

Our second hypothesis is that the demand of capital goods, measured by investment rate, can affect the manufacturing employment share. This question is addressed by estimating equations of the following variety

$$N_{2,i,t} = b_0 + b_1 \cdot \Delta A_{i,t} + b_2 \cdot Inv_{i,t} + b_3 \cdot Z_{i,t} + \epsilon_t. \tag{45}$$

where $N_{2,i,t}$ is the employment share of the manufacturing sector. The coefficient of investment, b_2, reflects whether there is an interaction between investment rate and manufacturing employment share. A positive and significant b_2 supports the view that investment demand determines manufacturing employment.

We construct a panel data covering 34 countries from 1950 to 2005. Most sectoral employment shares are calculated using the GGDC 10-sector database, while the national account data are from the Penn World Table (version 6.3).[20]

Table 4 presents the estimation results of equation (44). All regressions include change of agriculture employment as a covariate. We add output growth rate and real income per capita as control variables. The results confirm that there is a negative and significant relationship between the change of agriculture employment and investment, and the direction and magnitude of this coefficient is relatively stable as we add more control variables. The estimated a_1 range from -0.865 to -0.697, and are highly significant. It implies that a larger decrease of employment share in the agriculture sector is associated with

[19] According to our model, $N_{A,t} = N_{0,t} + N_{1,t} = \frac{\bar{C}_1}{B_1 k_t^{\alpha} X_t} + \alpha N_{0,t}$, thus $N_{A,t} - N_{A,t-1} \approx \alpha(N_{0,t} - N_{0,t-1})$.

[20] Data sources are listed in the Appendix, section B.

Table 4: Investment rate and structural change in a sample of 34 countries

	Dependent variable: Investment rate			
	(1)	(2)	(3)	(4)
Change of agriculture employment	-0.865***	-0.732***	-0.744***	-0.697***
	(0.211)	(0.207)	(0.176)	(0.182)
Manufacturing employment				0.238***
				(0.066)
Output growth rate		0.142***	0.140***	0.136***
		(0.041)	(0.043)	(0.046)
Income per capita			0.065***	0.046***
			(0.016)	(0.016)
Country dummies	Y	Y	Y	Y
Time dummies	Y	Y	Y	Y
No. of obs.	1350	1349	1349	1349
Adj.R^2	0.604	0.611	0.668	0.700

Notes: All regressions are OLS. Standard errors in parentheses. Robust standard errors are clustered at country level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 5: Manufacturing employment and investment rate in a sample of 34 countries

	Dependent variable: Manufacture employment				
	(1)	(2)	(3)	(4)	(5)
Change of agriculture employment	-0.291	0.256	0.210	0.143	0.130
	(0.239)	(0.252)	(0.254)	(0.198)	(0.195)
Investment rate		0.527***	0.533***	0.407***	0.409***
		(0.101)	(0.104)	(0.070)	(0.073)
Output growth rate			-0.054		
			(0.075)		
Income per capita				0.054*	0.096***
				(0.029)	(0.035)
Income per capita squared					-0.002*
					(0.001)
Country dummies	Y	Y	Y	Y	Y
Time dummies	Y	Y	Y	Y	Y
No. of obs.	1589	1350	1349	1350	1313
Adj.R^2	0.719	0.756	0.755	0.771	0.777

Notes: All regressions are OLS. Standard errors in parentheses. Robust standard errors are clustered at country level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

a higher share of resource to be invested. In addition, the regression results show that rapid economic growth and high-income level are also associated with high level of investment. Finally, the manufacturing employment share is positively associated with the investment rate, which has to be further investigated.

The main results for the determination of employment share in the manufacturing sector are summarized by Table 5. The results suggest that the investment rate is positively correlated with the employment share in the manufacturing sector, as the estimated b_2 range from 0.407 to 0.527 and are statistically significant at 1%, while the coefficients for the change of agriculture employment are insignificant. This result is stable for different sets of control variables.[21]

These regression results show that the decline of agriculture employment causes investment rate to increase. And changes of investment influence the manufacturing employment shares. This is an important piece of evidence that supports the mechanism proposed by our model.

6.2 The Peak Moment of the Manufacturing Employment

Both the theoretical analysis and numerical example show that the peak of the manufacturing employment during economic development, which is a main feature of structural transformation, can be a clear indicator for the adoption of modern agriculture technology. The rising manufacturing sector is associated with industrialization, during which modern society is transformed from agrarian societies as peasants became factory workers. After reaching the peak, the manufacturing sector completes its historical role and cedes its place to services.

Although the peak of manufacturing employment can be theoretically identified, it is challenging to be determined in the data, because the manufacturing employment share can be interrupted by varies external shocks, including, but not limited to, world wars, government policies, and severe economic crises. For example, in Malaysia and Thailand, their manufacturing employment shares reached all time highs in 1996 and 1997 before they were struck by the Asian

[21] To check for robustness, we use lagged variables as instrument variable, the results are similar to those reported by Tables 4 and 5.

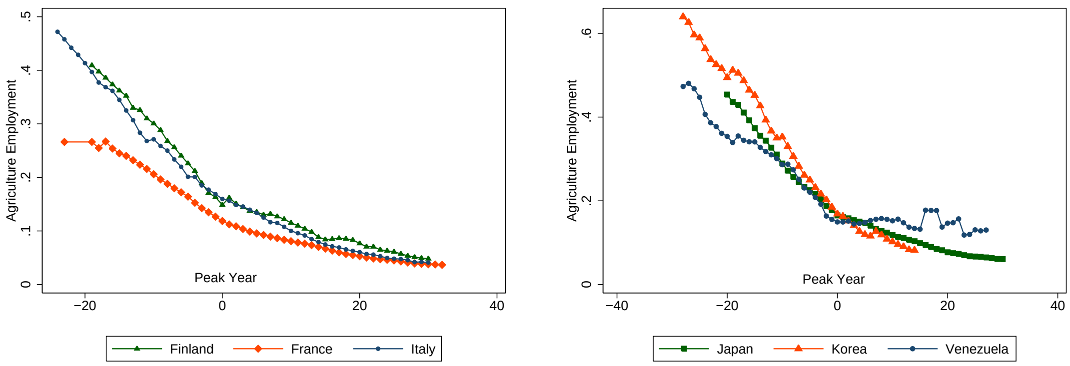
Table 6: Moments with peak manufacturing employments

Country	Code	Year	Income 2000$	Employment share Agriculture	Employment share Manufacturing
Developed country					
Australia	AUS	1964	15061	0.103	0.399
Austria	AUT	1966	13260	0.205	0.413
Canada	CAN	1956	13200	0.164	0.347
Denmark	DNK	1964	14642	0.155	0.370
Spain	ESP	1977	15377	0.187	0.354
Finland	FIN	1975	16610	0.149	0.361
France	FRA	1973	17878	0.119	0.361
Hong Kong	HKG	1976	9859	0.027	0.514
Italy	ITA	1975	16077	0.160	0.385
Japan	JPN	1973	17476	0.165	0.362
Korea	KOR	1991	12460	0.169	0.359
Netherlands	NLD	1965	15608	0.082	0.373
New Zealand	NZL	1967	15534	0.132	0.384
Singapore	SGP	1984	18187	0.013	0.387
Sweden	SWE	1965	16404	0.103	0.411
Taiwan	TWN	1987	11417	0.156	0.424
United Kingdom	GBR	1955	11926	0.046	0.458
United States	USA	1953	14916	0.073	0.335
Developing country					
Argentina	ARG	1958	6145	0.223	0.336
Brazil	BRA	1981	7381	0.326	0.243
Chile	CHL	1993	7002	0.169	0.265
Colombia	COL	1995	5271	0.265	0.209
Costa Rica	CRI	1994	8158	0.231	0.289
India	IND	2002	1980	0.610	0.169
Mexico	MEX	2000	10570	0.168	0.282
Malaysia	MYS	1997	9730	0.155	0.379
Peru	PER	1974	5831	0.459	0.205
Philippines	PHL	1997	2228	0.400	0.164
Venezuela	VEN	1978	11032	0.150	0.280

Sources: Groningen Growth and Development Centre (GGDC) 10-sector Historical National Accounts database.

financial crisis. After the crisis, Malaysia experienced a quick rebound and almost went back to the peak employment share in 2001, before it started to de-industrialize. In Thailand, the manufacturing employment recovered slowly from the crisis, rose to the pre-crisis level in 2005, and has continued to rise slightly since then. Therefore, in these cases, we recognize that Malaysia might have reached its peak moment in 2001 (the second and last high employment share in manufacturing before the de-industrialization); and we treat Thailand as an emerging economy in which the manufacturing employment share still has potential to increase. The same logic is applied to other developing countries.

Using the above criterion, we identify the peaks of manufacturing employment shares in 29 countries, as summarized in Table 6. It shows the developed countries today share very similar experience of structural transformation: they have high peak employment shares in the manufacturing sector at higher income levels. And, most of their agriculture employment shares were already less than 20% at the peak. In addition, 11 economies out of 29 in our sample had their agriculture share in the small range between 15% and 17%. And only 5 economies had less than 10% of workers in the agriculture sector before the employment share of the manufacturing sector declined, including Netherlands, U.K., U.S., and two city states, Hong Kong and Singapore. Thus, the structural transformation of the United States, despite its popularity in the literature, is somehow an exception.



(a) Finland, France, and Italy (b) Japan, Korea, and Venezuela

Source: Various historical statistics, see section B in the Appendix.

Figure 8: Agricultural employment shares before and after the peak year of manufacturing employment

With the help of these estimated peak moments, we can break any structural transformation process into two sub-periods, before and after the manufacturing peak, which is set to be a common moment across countries. Both the theoretical and numerical analysis in the previous sections predict that the employment share of the agriculture sector decreases faster before reaching the peak moment. Figure 8 shows that in selected countries, the rate of decline for the agriculture sector does slow down after the manufacturing employment share has peaked.

6.3 Agriculture Modernization and Economic Growth

We finally turn to the relationship between agriculture modernization and economic growth. Our model predicts that the change of employment share in the agriculture sector is associated with economic growth. For example, according to proposition 4, in the mixed stage, the output growth rate, $g_{Y,t} = \frac{Y_{t+1}-Y_t}{Y_t}$, satisfies

$$g_{Y,t} = -\frac{\alpha \Delta N_{0,t}}{1-\alpha N_{0,t}} = -\frac{\Delta N_{A,t}}{1-\alpha N_{0,t}}. \tag{46}$$

And in the convergent stage and along the GBGP, according to equations (24) and (41), the relationship between $g_{Y,t}$ and $\Delta N_{A,t}$ is provided by

$$g_{Y,t} = -\frac{\Delta N_{A,t}}{N_{A,t}}. \tag{47}$$

Therefore, we expect that change of agriculture employment share is negatively correlated with economic growth rate. To test this formally, we consider the following empirical specification

$$g_{Y,i,t} = d_0 + d_1 \cdot \Delta A_{i,t} + d_2 \cdot Z_{i,t} + \epsilon_t, \tag{48}$$

where $\Delta A_{i,t} = \Delta N_{A,t} = N_{A,i,t} - N_{A,i,t-1}$, and $Z_{i,t}$ represents control variables.

Table 7 shows a set of regressions in which growth rate is regressed on the change of agriculture employment and other control variables. Columns (1) and (2) show that the correlation between the change of employment shares in the agriculture sector is significantly negative with output growth rate. In addition, using the identified peak moments of manufacturing employment, we construct a dummy variable called "post peak indicator", which is 0 before the peak and is 1 since reaching the peak. Therefore, this post peak indicator identifies the mixed stage and the convergent stage discussed in our theoretic framework. Column (3) shows that the estimated coefficient of this post peak indicator is -0.013, which is significant at 1% level. Considering the average growth rate in our sample is about 0.026, this slowdown of output growth is very significant, and is consistent with our model prediction (see Figure 4).

Table 7: Agriculture modernization and structural change in a sample of 34 countries

	Dependent variable: growth rate		
	(1)	(2)	(3)
Change of agriculture employment	-1.058***	-0.817***	-0.737***
	(0.198)	(0.177)	(0.176)
Post peak indicator			-0.013***
			(0.004)
Country dummies	N	Y	Y
Time dummies	N	Y	Y
No. of obs.	1567	1567	1567
Adj.R^2	0.067	0.252	0.260

Notes: All regressions are OLS. Standard errors in parentheses. Robust standard errors are clustered at country level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

7 Concluding Remarks

This paper introduces a new mechanism, agriculture modernization, to explain the structural change during economic development. Based on the fact that the factor input structures in the agriculture sector are significantly different between rich and poor countries, we assume that there is technology switch for producing food during economic development, which is called agriculture modernization.

The economic intuition can be briefly summarized as the following: a traditional agriculture sector relies on raw labor input, whereas modern agriculture production utilizes physical capital. Productivity improvement in the modern sectors eventually triggers the transition to adopt modern technology. As workers move from traditional agriculture sector to modern sectors, the demand for capital goods raises investment rate. Since capital goods are supplied by the manufacturing sector, the manufacturing output and employment expand. As this agriculture modernization completes, both investment rate and manufacturing employment decrease, and the whole economy would converge to a generalized balanced growth path.

We adopt a simulation approach to illustrate the dynamic features of our model. Unlike the existing literature, our model is able to generate hump-shaped patterns for manufacturing employment shares, investment rates, and economic growth rates, without assuming unbalanced technology growth. These results provide a common mechanism to explain the observed universal pattern of structural transformation during long-term economic growth.

Three pieces of empirical evidence are provided to support our theory. We start by testing the primary mechanism, which links agriculture modernization with structural transformation through investment. Using a panel data covering 34 countries from 1950 to 2005, we find that the decline of agriculture employment can cause investment rate to increase, but can not affect the labor employment share in the manufacturing sector, while investment rate movements are highly correlated with employment change in the manufacturing sector.

We supplement this argument by identifying the highest level of manufacturing employment share for countries in our dataset, which performs as an indicator and marks the end of agriculture modernization in our model. We argue that the decline of agriculture employment slows down in this post peak period.

Finally, we find that, consistent with our model, the link between agriculture modernization and economic growth is statistically significant.

Appendix

A Mathematical Details

Proposition. 1 *If we let* $Z_t = \left(\frac{B_0}{(1-\alpha)B_1}\right)^{\frac{1}{\alpha}} X_t^{-\frac{1}{\alpha}}$*, in agriculture production, the firm*

1) uses only traditional technology, if $k_t < Z_t$*;*

2) uses only modern technology, if $k_t > Z_t$*;*

3) and uses a mixed combination, if $k_t = Z_t$*.*

Proof. The cost of per unit of agriculture product using the traditional technology is given by

$$Cost_{0,t} = \frac{W_t}{B_{0,t}}.$$

The modern technology gives the cost function as the following

$$Cost_{1,t} = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}} \frac{R_t^\alpha}{B_{1,t}} \left(\frac{W_t}{X_t}\right)^{1-\alpha}.$$

If both technologies are equally cost efficient, we have the mixed production condition,

$$k_t = \left(\frac{B_{0,t}}{(1-\alpha)X_t B_{1,t}}\right)^{\frac{1}{\alpha}},$$

where $k_t = \frac{K_t}{(1-N_{0,t})X_t}$ is the capital/labor ratio in the modern sector.

The food production uses only traditional technology if

$$k_t \leqslant \left(\frac{B_{0,t}}{(1-\alpha)X_t B_{1,t}}\right)^{\frac{1}{\alpha}};$$

and uses only modern technology if

$$k_t \geqslant \left(\frac{B_{0,t}}{(1-\alpha)X_t B_{1,t}}\right)^{\frac{1}{\alpha}}.$$

□

Proposition. 2 *Whenever assumption 2 holds,* $\frac{\bar{C}_1}{\bar{C}_3} = \frac{B_1}{B_3}$*, a generalized balanced growth path exists. The relative prices, aggregate labor income share, and growth rate of output and capital are constant. The employment share declines in agriculture, rises in services, and is stable in manufacturing. The capital rental rate and sectoral capital per effective labor consistent with this generalized balanced growth path are given by*

$$\bar{r} = \frac{1+g}{\beta} + \delta - 1.$$

$$\bar{k} = \left(\frac{\alpha B_2}{\bar{r}}\right)^{\frac{1}{1-\alpha}}$$

Proof. Assumption 2 yields $p_1\bar{C}_1 = p_3\bar{C}_3$. Thus, using the result of optimal consumption, equation (14), we can we rewrite the resources constraint for the modern economy, equation (13), as

$$\begin{aligned} B_2 k_t^\alpha X_t &= p_1 \bar{C}_1 + I_t + C_{2,t} + p_3 C_{3,t}, \\ &= I_t + C_{2,t} + p_3 (C_{3,t} + \bar{C}_3), \\ &= I_t + \frac{C_{2,t}}{\gamma}. \end{aligned}$$

There exists a steady-state level of capital/labor ratio $k_t = \bar{k}$, which implies that the left-hand side expands at a constant rate g. On the right-hand side, both investment (I_t) and consumption aggregation ($\frac{C_{2,t}}{\gamma}$) can also grow at rate g. The corresponding capital rental rate $\bar{r} = \frac{1+g}{\beta} + \delta - 1$. And $\bar{k} = \left(\frac{\alpha B_2}{\bar{r}}\right)^{\frac{1}{1-\alpha}}$.

The economy is on a generalized balanced growth path. □

Proposition. 3 *In the stage of mixed economy, for any time $t > M$, if $N_{0,t} > 0$, $N_{1,t} > 0$, thus*

$$k_t = \left[\frac{B_0}{B_1(1-\alpha)X_t}\right]^{\frac{1}{\alpha}}, \tag{49}$$

where k_t satisfies $k_t = \frac{k_{t-1}}{(1+g)^{1/\alpha}} < k_{t-1} \leqslant k_M$.

Proof. Since both types of technology share the same relative price in the mixed economy, for $t > M$, $p_{0,t} = p_1$ gives $k_t = \left[\frac{B_0}{B_1(1-\alpha)X_t}\right]^{\frac{1}{\alpha}}$, where $k_t < k_{t-1}$ as $X_t > X_{t-1}$. □

Proposition. 4 *The total output of this economy in the mixed stage is determined by the process of agriculture modernization,*

$$Y_t = \frac{B_2 B_0}{B_1(1-\alpha)}(1 - \alpha N_{0,t}); \tag{50}$$

The investment rate in the mixed stage satisfies

$$iy_t = \frac{\left[(1 - N_{0,t+1})(1+g)^{\frac{\alpha-1}{\alpha}} - (1-\delta)(1-N_{0,t})\right]}{B_2(1-\alpha N_{0,t})}\left(\frac{B_0}{(1-\alpha)B_1 X_t}\right)^{\frac{1-\alpha}{\alpha}}. \tag{51}$$

Proof. The total output is the sum of products from all sectors,

$$\begin{aligned} Y_t &= \sum_{i \in \{0,1,2,3\}} p_{i,t} Y_{i,t}, \\ &= \frac{B_2}{B_1} B_0 N_{0,t} + B_2 k_t^\alpha (1 - N_{0,t}) X_t. \end{aligned}$$

Since $k_t = \left[\frac{B_0}{B_1(1-\alpha)X_t}\right]^{\frac{1}{\alpha}}$, we have

$$\begin{aligned} Y_t &= \frac{B_2}{B_1} B_0 N_{0,t} + \frac{B_2 B_0}{B_1(1-\alpha)}(1-N_{0,t}), \\ &= \frac{B_2 B_0}{B_1(1-\alpha)}(1-\alpha N_{0,t}). \end{aligned}$$

Investment function is given by

$$\begin{aligned} I_t &= K_{t+1} - (1-\delta)K_t \\ &= \left[k_{t+1}(1-N_{0,t+1})(1+g) - (1-\delta)k_t(1-N_{0,t})\right] X_t. \end{aligned}$$

Since $k_t = \left(\frac{B_0}{(1-\alpha)X_t B_1}\right)^{\frac{1}{\alpha}}$ in the mixed stage, simple derivation yields

$$\begin{aligned} I_t &= \left[(1-N_{0,t+1})(1+g)^{\frac{\alpha-1}{\alpha}} - (1-\delta)(1-N_{0,t})\right] X_t k_t, \\ &= \left[(1-N_{0,t+1})(1+g)^{\frac{\alpha-1}{\alpha}} - (1-\delta)(1-N_{0,t})\right] \left(\frac{B_0}{(1-\alpha)B_1}\right)^{\frac{1}{\alpha}} X_t^{\frac{\alpha-1}{\alpha}}. \end{aligned}$$

Therefore, the investment rate, $iy_t = \frac{I_t}{Y_t}$, is given by

$$\begin{aligned} iy_i &= \frac{\left[(1-N_{0,t+1})(1+g)^{\frac{\alpha-1}{\alpha}} - (1-\delta)(1-N_{0,t})\right] \left(\frac{B_0}{(1-\alpha)B_1}\right)^{\frac{1}{\alpha}} X_t^{\frac{\alpha-1}{\alpha}}}{\frac{B_2 B_0}{B_1(1-\alpha)}(1-\alpha N_{0,t})}, \\ &= \frac{\left[(1-N_{0,t+1})(1+g)^{\frac{\alpha-1}{\alpha}} - (1-\delta)(1-N_{0,t})\right]}{B_2(1-\alpha N_{0,t})} \left(\frac{B_0}{(1-\alpha)B_1 X_t}\right)^{\frac{1-\alpha}{\alpha}}. \end{aligned}$$

□

Proposition. 5 *The movements of employment shares in the mixed economy exhibit the following properties:*

1) Total employment shares used to produce agriculture goods start to decline,

$$N_{A,t+1} - N_{A,t-1} = \alpha(N_{0,t+1} - N_{0,t});$$

2) The size of the traditional agriculture sector, in terms of employment, is given by

$$\frac{G(N_{0,t+1})}{G(N_{0,t})} = \beta(\alpha B_2 k_t^{\alpha-1} + 1 - \delta),$$

where k_t is given by equation (28), and $G(N_{0,t+1}) = C_{2,t}$ satisfies

$$\begin{aligned} G(N_{0,t+1}) \;\; &\equiv \;\; -\gamma \left[\frac{1 - N_{0,t+1}}{(1+g)^{\frac{1-\alpha}{\alpha}}} - (1-\delta)(1 - N_{0,t}) \right] \left(\frac{B_0}{(1-\alpha)B_1} \right)^{\frac{1}{\alpha}} X_t^{\frac{\alpha-1}{\alpha}} \\ &+ \frac{\gamma B_2 B_0}{B_1} \frac{1 - \alpha N_{0,t}}{1-\alpha}; \end{aligned}$$

3) The employment shares in the manufacturing and service sectors are given by

$$N_{2,t} = \gamma(1-\alpha N_{0,t}) + \frac{1-\gamma}{B_2} \left[\frac{1 - N_{0,t+1}}{(1+g)^{\frac{1-\alpha}{\alpha}}} - (1-\delta)(1 - N_{0,t}) \right] \left(\frac{B_0}{(1-\alpha)X_t B_1} \right)^{\frac{1-\alpha}{\alpha}},$$

$$N_{3,t} = \frac{B_1(1-\alpha)}{B_0 B_3} \left[\frac{1-\gamma}{\gamma} \frac{G(N_{0,t+1})}{p_3} - \bar{C}_3 \right].$$

Proof. 1) In the mixed economy, $k_t = Z_t$, let n_1 and n_0 denote the labor inputs used to produce one unit of agriculture goods, we have

$$n_1 = \frac{k_t^{-\alpha}}{B_1 X_t} = \frac{Z_t^{-\alpha}}{B_1 X_t} = \frac{1-\alpha}{B_0} < \frac{1}{B_0} = n_0.$$

Therefore, the modern technology uses less labor input to produce one unit of agriculture goods. Because the total consumption for agriculture goods is fixed at $\bar{C}_1$, the adoption of modern production will release labor from the traditional sector and decrease the total employment share in the agriculture sector.

The aggregate employment share to produce food is $N_{A,t} = N_{0,t} + N_{1,t} = \frac{\bar{C}_1}{B_1 k_t^{\alpha} X_t} + \alpha N_{0,t}$. In the mixed stage, $\frac{\bar{C}_1}{B_1 k_t^{\alpha} X_t} = \frac{1-\alpha}{B_0}\bar{C}_1$, thus $N_{A,t+1} - N_{A,t} = \alpha(N_{0,t+1} - N_{0,t})$.

2) Since $k_t = Z_t$, the relative price, equation (12), yields $p_{0,t} = \frac{B_2}{B_0}(1-\alpha) k_t^{\alpha} X_t = \frac{B_2}{B_1} = p_1$.

The resource constraint of this economy, equation (13), can be characterized by

$$\begin{aligned}
B_2 k_t^{\alpha}(1-N_{0,t})X_t &= I_t + \frac{C_{2,t}}{\gamma} - \frac{B_2 B_0}{B_1} N_{0,t}, \\
&= \left[\frac{1-N_{0,t+1}}{(1+g)^{\frac{1-\alpha}{\alpha}}} - (1-\delta)(1-N_{0,t})\right] k_t X_t \\
&\quad + \frac{C_{2,t}}{\gamma} - \frac{B_2 B_0}{B_1} N_{0,t}. \qquad (52)
\end{aligned}$$

Equation (52) suggests that $C_{2,t}$ is a function of $N_{0,t+1}$,

$$\begin{aligned}
C_{2,t} &= \gamma\left[B_2 k_t^{\alpha}(1-N_{0,t})X_t + \frac{B_2 B_0}{B_1} N_{0,t}\right] - \gamma\left[\frac{1-N_{0,t+1}}{(1+g)^{\frac{1-\alpha}{\alpha}}} - (1-\delta)(1-N_{0,t})\right] k_t X_t, \\
&= \frac{\gamma B_2 B_0}{B_1}\frac{1-\alpha N_{0,t}}{1-\alpha} - \gamma\left[\frac{1-N_{0,t+1}}{(1+g)^{\frac{1-\alpha}{\alpha}}} - (1-\delta)(1-N_{0,t})\right]\left(\frac{B_0}{(1-\alpha)B_1}\right)^{\frac{1}{\alpha}} X_t^{\frac{\alpha-1}{\alpha}} \\
&\equiv G(N_{0,t+1}).
\end{aligned}$$

Plugging in the intertemporal Euler equation, equation (15), the dynamic of this economy is given by

$$\frac{C_{2,t}}{C_{2,t-1}} = \frac{G(N_{0,t+1})}{G(N_{0,t})} = \beta(\alpha B_2 k_t^{\alpha-1} + 1 - \delta).$$

3) Combining equations (17) and (18), the labor employment in the manufacturing sector is given by

$$\begin{aligned}
N_{2,t} &= \frac{C_{2,t} + I_t}{B_2 k_t^{\alpha} X_t}, \\
&= \gamma(1-N_{0,t}) + \frac{\gamma}{k_t^{\alpha} X_t}\left(\frac{B_0}{B_1} N_{0,t} + \frac{\bar{C}_3}{B_3} - \frac{\bar{C}_1}{B_1}\right) \\
&\quad + \frac{1-\gamma}{B_2}\left[\frac{1-N_{0,t+1}}{(1+g)^{\frac{1-\alpha}{\alpha}}} - (1-\delta)(1-N_{0,t})\right] k_t^{1-\alpha}.
\end{aligned}$$

Since $k_t = \left(\frac{B_0}{(1-\alpha)X_t B_1}\right)^{\frac{1}{\alpha}}$ in the mixed stage and $\frac{\bar{C}_1}{\bar{C}_3} = \frac{B_1}{B_3}$ (assumption 2), we have

$$N_{2,t} = \gamma(1-\alpha N_{0,t}) + \frac{1-\gamma}{B_2}\left[\frac{1-N_{0,t+1}}{(1+g)^{\frac{1-\alpha}{\alpha}}} - (1-\delta)(1-N_{0,t})\right]\left(\frac{B_0}{(1-\alpha)X_t B_1}\right)^{\frac{1-\alpha}{\alpha}},$$

$$\begin{aligned} N_{3,t} &= \frac{C_{3,t}}{B_3 k_t^\alpha X_t} = \frac{1}{B_3 k_t^\alpha X_t}\left[\frac{1-\gamma}{\gamma}\frac{G(N_{0,t+1})}{p_3} - \bar{C}_3\right], \\ &= \frac{B_1(1-\alpha)}{B_0 B_3}\left[\frac{1-\gamma}{\gamma}\frac{G(N_{0,t+1})}{p_3} - \bar{C}_3\right]. \end{aligned}$$

□

Proposition. 6 *In the convergent stage, the employment share of the manufacturing sector decreases. If k_t converges along the saddle path to $\bar{k}$, the manufacturing employment share also moves towards $N_{2,G}$ on the generalized balanced growth path.*

Proof. According to assumption 4.3, $k_C < \bar{k}$. k_t evolves on a saddle path and converges to the steady state, for $t > C$, we have $k_{t+1} > k_t$.

For the manufacturing employment,

$$N_{2,t+1} - N_{2,t} = \frac{1-\gamma}{B_2}\left[(1+g)\left(\frac{k_{t+2}}{k_{t+1}^\alpha} - \frac{k_{t+1}}{k_t^\alpha}\right) + (1-\delta)\left(k_t^{1-\alpha} - k_{t+1}^{1-\alpha}\right)\right].$$

We rewrite the Euler equation

$$\frac{B_2 k_{t+1}^{\alpha-1} - \left[\frac{k_{t+2}}{k_{t+1}}(1+g) - (1-\delta)\right]}{B_2 k_t^{\alpha-1} - \left[\frac{k_{t+1}}{k_t}(1+g) - (1-\delta)\right]} = \frac{\beta(r_{t+1} + 1 - \delta)}{1+g} > 1,$$

which gives

$$(1+g)\left(\frac{k_{t+2}}{k_{t+1}} - \frac{k_{t+1}}{k_t}\right) < B_2\left(k_{t+1}^{\alpha-1} - k_t^{\alpha-1}\right) < 0.$$

Since $k_t^{1-\alpha} - k_{t+1}^{1-\alpha} < 0$, we have $N_{2,t+1} < N_{2,t}$.

In the convergent stage, if k_t rises along the saddle path toward $\bar{k}$, from equation (42), we have $N_{2,t}$ converge to

$$\bar{N}_{2,G} = \gamma + \frac{1-\gamma}{B_2}(g+\delta)\bar{k}.$$

□

B Data Sources

Our main source of data is the Groningen Growth and Development Centre 10-sector data (Timmer and Vries, 2008).[22] It covers 33 countries from 1950 to 2005. We add the sectoral employment shares for China using the China Statistical Yearbook (2011). Therefore, our sample overall includes 34 countries.[23] The latest update available for each country was used. Data for Latin American and Asian countries came from the June 2007 update, while data for the European countries and the United States came from the October 2008 update.

The three broad sectors are categorized as the following: the primary sector (agriculture), which only includes agricultural production; the secondary sector (manufacturing), which consists of mining, manufacturing, public utilities and construction; the tertiary sector (service), which covers wholesale, retail trade (including hotels and restaurants), transport, storage, and communication finance, insurance, and real estate and community, social and personal services, and government services.

The real GDP per capita comes from the Penn World Table (version 6.3), while BEA reports investment to output ratio, and capital to output ratio. The ratio of investment to output comes from Kuznets (1966) Table 5.5 and the World Development Index by the World Bank.

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[22]Available at http://www.ggdc.net/ databases/10_sector.htm

[23]The complete country list includes, Argentina, Australia, Austria, Bolivia, Brazil, Canada, Chile, China, Colombia, Costa Rica, Denmark, Spain, Finland, France, United Kingdom, Germany, Hong Kong, Indonesia, India, Italy, Japan, Korea, Mexico, Malaysia, Netherlands, New Zealand, Peru, Philippines, Singapore, Sweden, Thailand, Taiwan, United States, and Venezuela.

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