Capital Goods Import, Balance of Payments Constraint, and Economic Growth*

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Abstract

This paper explores the interaction between trade and growth. In particular, we assume that the information of advanced technology is embodied within high-quality capital goods, which are produced by developed economies. Thus, the international technology diffusion goes through the channel of trading high quality capital goods, which establishes a direct causal linkage from trade to growth. The capital import is subject to a balance of payment constraint, and must be financed by exports. We define and characterize two types of steady states and illustrate the interaction between the balance of payments constraint and the optimal capital import condition.

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1 Introduction

The model presented in this paper intends to explore the interaction between trade and growth. We assume that importing foreign capital goods can improve the quality of domestic capital stock and product, because the information of advanced technology is embodied within high-quality capital goods produced by developed countries. Therefore, capital goods importing is an important channel of international technology diffusion, which establishes a direct causal linkage from trade to growth.

This model consolidates existing evidences into one explanation. It shows that the observed export expansion is used to finance the import of foreign capital goods to achieve quality upgrading. And the industry upgrading process can be summarized by this process of quality improvement. Since the output growth is primarily caused by the used of high-quality capital goods with embodied technology, the standard growth accounting exercise might overstate the role of capital accumulation in economic growth.

As trade pattern is subject to the balance of payments constraint, foreign demand, term of trade, and trade balance can significantly affect economic growth. For developing countries, our framework indicates two barriers for economic growth. The first barrier is that the feasibility of trade can be restricted by the low product quality, because high-quality capital goods are only produced by a few rich countries who trade intensively with each other (Eaton and Kortum, 2001; Hallak, 2006). Thus, in order to exchange sufficient capital goods, developing countries have to compete for the limited opportunities to export. However, if a developing country decides to stimulate foreign demand through currency depreciation, the second barrier arises, because the foreign capital might become too expensive to be imported for investment. As a result, our model shows that the condition for economic growth can be very tricky. Depending on different trade patterns, both convergence and divergence of income level could take place.

Our model is supported by several strands of literature. Our explanation of the export expansion during economic growth coincides with the main idea of Rodrik (1997), though the cause of import increase is different. In our model, importing foreign capital goods plays as the primary channel of international technology diffusion, while Rodrik (1997) argued that the profitability of domestic investment raises the demand for capital goods and increases capital import consequently. Our emphasis on the balance of payments constraint is distinct from the orthodox growth theory, which links us to the balance of payments constrained growth models.[1] Thirlwall (2011) claimed that "in the long run, no country can grow faster than that rate consistent with balance of payments equilibrium on current account unless it can finance ever-growing deficits which, in general, it cannot." Therefore, in our baseline model, a crucial condition for achieving sustainable growth is to maintain a non-negative current account.

The role of imported capital goods on growth has strong empirical roots. Lee (1995) used cross-country data for the period 1960 to 1985 and showed that the ratio of imported capital goods to domestically produced capital goods in the composition of investment is positively related with the per capita income growth rate. In a recent study, Herrerias and Orts (2013) confirmed that the ratio of imported to domestic capital goods determined the long-run growth rate and argue that the link between trade openness and long-run growth operates mainly through imports.

We owe a major intellectual debt to the growing literature on product quality. Since rich countries import more and consume more from countries producing high-quality goods (Hallak, 2006), we assume that low product quality dampens demand. In addition, Feenstra and Romalis (2014) found that the exports of rich countries tend to be of high quality, whereas poor countries tend to have notably lower quality exports. Thus, in our model, the primary channel of income convergence relies on product quality convergence. Henn et al. (2013) found out that quality upgrading is particularly rapid during the early stages of development, with quality convergence largely completed as a country reaches upper middle-income status, and the quality upgrading was particularly impressive in East Asia.

The rest of this paper proceeds as follows. Section 2 presents the basic economic environment of our two-country model, in which the primary channel of technology improvement is importing high-quality capital goods. We characterize the economic equilibrium in section 3 and define two types of balanced growth paths in section 4. Section 5 discusses an extension of the basic model and two related implications. And section 6 concludes.

2 A Simple Model

We start our analysis with a simple two-country model. Home country, H, is a low-income country, while foreign country, F, is a developed country. In general, we will

[1] See Thirlwall and Hussain (1982), Thirlwall (2011), and many others.

mark foreign variables with asterisks.

2.1 Production

For the sake of simplicity, we assume that these two economies are very similar to each other. Each country produces one product that can be used for both consumption and investment, with the following production technologies

$$Y_t = (Z_t)^{\alpha} (N_t)^{1-\alpha}, \; Y_t^* = (Z_t^*)^{\alpha} (N_t^*)^{1-\alpha}, \tag{1}$$

where N_t and N_t^* are labor inputs, and Z_t and Z_t^* are capital stocks in in efficiency units. The law of motion for Z_t is given by

$$\dot{Z}_t = H_t - \delta Z_t,$$

where H_t is investment measured in efficiency units.

$Z_t = q_t K_t$ and $Z_t^* = q_t^* K_t^*$, which are described by two types of variables, the quality indexes, q_t and q_t^*, reflecting the production efficiency of capital goods, and K_t and K_t^* representing the quantities of capital goods. Therefore, the change of Z_t can be divided into changes in two dimensions: the change of capital quality, and the change of capital quantity,

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{q}_t}{q_t} + \frac{\dot{K}_t}{K_t}. \tag{2}$$

The law of motions for the quantity of capital stock are given by

$$\dot{K}_t^* = I_{F,t}^* - \delta K_t^*, \tag{3}$$

$$\dot{K}_t = I_{H,t} + I_{F,t} - \delta K_t, \tag{4}$$

where $I_{H,t}$, $I_{F,t}$, and $I_{F,t}^*$ represent the capital goods used by the home country that are produced by the home country, capital goods used by the home country that are produced by the foreign country, and foreign-produced capital goods used by the foreign country, respectively.[2] Since they represent investment flows, they are assumed to be non-negative.

The quality improvement mechanisms are different across countries. Since the for-

[2] Here, we assume that the foreign capital goods and domestic capital goods are perfect substitutes in terms of quantity, as the quality properties have been represented by q.

eign country is considered to be the leader of technology innovation on the frontier, the product quality of the foreign country is assumed to improve at an exogenous constant rate g, which can only be partially captured by the developing home country. However, the home country can import foreign-produced capital goods to improve the quality of its capital stock. Thus, the quality improvement progress is given by

$$\hat{q}_t^* = \frac{\dot{q}_t^*}{q_t^*} = g, \tag{5}$$

$$\hat{q}_t = Q_t g + \phi_t(\frac{1}{Q} - 1), \tag{6}$$

where Q_t is defined as a relative quality index

$$Q_t = \frac{q_t}{q_t^*}, \text{ and } 0 < Q_t \leqslant 1, \tag{7}$$

which measures the efficiency difference of capital stocks between the home country and the foreign country. For $Q_t = 1$, the production functions are the same for both countries, and for $Q_t < 1$, the home production is less efficient than the foreign production.

The first component on the right-hand side of equation (6) describes the benefit of the home country from the foreign technology innovation as a spillover effect. However, only a portion of the technology progress can be learned by them, since their relatively low quality of capital stock and production limits their benefits. The second component represents the channel of importing high-quality capital goods from the foreign country to improve the quality of production. However, the foreign capital goods used in developing countries are less efficient (in absolute term), since they have to cooperate with domestic low-quality capital goods, and can be adversely affected by weak local institutions. The quantity share of newly imported foreign capital goods in home countries capital stock is given by

$$\phi_t = \frac{I_{F,t}}{K_t}. \tag{8}$$

Without exogenous technology improvement ($g = 0$), we can show that our model is equivalent to Hulten (1992)'s vintage investment model with embodies technical change.

The law of motion for capital stock in efficiency units[3] satisfies

$$\begin{aligned}\dot{Z}_t &= q_t I_{H,t} + q_t^* I_{F,t} - \delta Z_t, \\ &= q_t(I_{H,t} + I_{F,t} - \delta K_t) + (q_t^* - q_t) I_{F,t}.\end{aligned}$$

The growth rate for capital stock is given by

$$\begin{aligned}\frac{\dot{Z}_t}{Z_t} &= \frac{q_t(I_{H,t} + I_{F,t} - \delta K_t)}{q_t K_t} + \frac{(q_t^* - q_t) I_{F,t}}{q_t K_t}, \\ &= \frac{I_{H,t} + I_{F,t} - \delta K_t}{K_t} + \left(\frac{1}{Q} - 1\right)\phi_t, \\ &= \frac{\dot{K}_t}{K_t} + \frac{\dot{q}_t}{q_t}.\end{aligned}$$

2.2 Preference

Each economy is populated by an infinitely lived representative family. For simplicity, we assume the family size grows at a constant rate n and $N_0 = N_0^*$,

$$N_t = N_0 e^{nt}. \tag{9}$$

The representative family of country i maximizes their lifetime utility as the following

$$\int_0^\infty \frac{C_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \tag{10}$$

where ρ is the rate of time preference (measure of impatience). And C_t is a comprehensive consumption index that depends on quality-adjusted consumption goods from

[3]Hulten (1992) used $Z_{t+1} = H_t + (1-\delta) Z_t$ to describe the law of motion for efficient capital stock, where $H_t = q_t I_{H,t} + q_t^* I_{F,t}$.

both home and foreign countries,[4]

$$C_t = \left[\gamma_H^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + (1-\gamma_H)^{\frac{1}{\eta}}\left(\frac{C_{F,t}}{Q_t^{\theta_H}}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{11}$$

$$C_t^* = \left[\gamma_F^{\frac{1}{\eta}}(C_{F,t}^*)^{\frac{\eta-1}{\eta}} + (1-\gamma_F)^{\frac{1}{\eta}}\left(Q_t^{\theta_F}C_{H,t}^*\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{12}$$

where γ_H and γ_F represent domestic preference weight on domestically produced consumption goods, we assume $\gamma_F \geqslant \gamma_H \geqslant \frac{1}{2}$. This assumption generates a home consumption bias. The elasticity of substitution between domestically produced and imported consumption goods is denoted by η, and we assume $\eta > 1$.[5]

In both countries, imported consumption goods are adjusted by the relative quality index Q_t with non-negative parameters, θ_H and θ_F. The economic intuition of this quality adjustment can be easily interpreted as we solve the household's optimization problem to derive the following consumption allocations

$$C_{F,t} = \frac{1-\gamma_H}{\gamma_H}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{\eta}\frac{C_{H,t}}{Q_t^{\theta_H(\eta-1)}} = (1-\gamma_H)\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\frac{C_t}{Q_t^{\theta_H(\eta-1)}}, \tag{13}$$

$$C_{H,t}^* = \frac{1-\gamma_F}{\gamma_F}\left(\frac{P_{F,t}^*}{P_{H,t}^*}\right)^{\eta}Q_t^{\theta_F(\eta-1)}C_{F,t}^* = (1-\gamma_F)\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta}Q_t^{\theta_F(\eta-1)}C_t^*, \tag{14}$$

where $P_{H,t}$ is the domestic price of home product in home country, $P_{H,t}^*$ is the foreign price of home product in foreign country, $P_{F,t}$ and $P_{F,t}^*$ represents the prices of foreign product in home and foreign countries, and P_t and P_t^* are consumption-based aggregate price indexes

$$P_t = \left[\gamma_H P_{H,t}^{1-\eta} + \frac{1-\gamma_H}{Q_t^{\theta_H(\eta-1)}}(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{15}$$

$$P_t^* = \left[\gamma_F\left(P_{F,t}^*\right)^{1-\eta} + (1-\gamma_F)\left(P_{H,t}^*\right)^{1-\eta}Q_t^{\theta_F(\eta-1)}\right]^{\frac{1}{1-\eta}}. \tag{16}$$

[4]These two consumption indexes are based on their local product, respectively. In order to compare the absolute consumption, we can easily transform them into indexes based on one common product, e.g. the foreign product.

[5]Obstfeld and Rogoff (2005, 2007) argued that, although the estimates of the trade elasticity cover a wide range, they typically include many values much higher than 2. Thus, they used 2 and 3 as representative values for aggregate trade elasticity. Examples of recent estimates can be found from Broda and Weinstein (2006).

Equations (13) and (14) describe the way that relative quality index affects consumption demand. For example, Given foreign consumption index C_t^*, equation (14) shows that a small Q_t leads to a weak foreign demand for the home country. Therefore, $Q_t^{\theta_F(\eta-1)} < 1$ performs as a quality punishment factor on home country's product, while $\frac{1}{Q_t^{\theta_H(\eta-1)}} > 1$ represents a quality premium from consuming foreign product. It shows that, given relative prices, households of foreign country prefer consumption variety, but only a relatively small amount of low-quality consumption goods from the home country are sufficient to make them satisfied. This implication is consistent with the empirical finding of Hallak (2006) that rich countries import more from countries producing high-quality goods.

The reason that the absolute quality levels, q_t and q_t^*, do not enter the consumption index is that q_t and q_t^* measure the quality of capital goods rather than the quality of consumption goods. Since they have already been used as efficiencies indexes in the production functions, including them in the consumption index causes a double counting problem and prevents us from solving the steady states.[6]

The resource constraint of the home country is given by

$$P_{H,t}(C_{H,t} + I_{H,t}) + P_{F,t}(C_{F,t} + I_{F,t}) = W_{H,t}N_t + R_{H,t}Z_t. \tag{17}$$

Similarly, the resource constraint of the foreign country is given by

$$P_{H,t}^*(C_{H,t}^*) + P_{F,t}^*(C_{F,t}^* + I_{F,t}^*) = W_{F,t}^*N_t^* + R_{F,t}^*Z_t^*. \tag{18}$$

2.3 Trade Balance and Market Clearing Conditions

The aggregate trade balance of home country, TB_t, is given by

$$TB_t = P_{H,t}C_{H,t}^* - P_{F,t}\left(I_{F,t} + C_{F,t}\right). \tag{19}$$

We assume the law of one price holds in our model. The nominal exchange rate ϵ_t is defined as the ratio of prices in home country to the prices in foreign country.

[6] Using absolute quality indexes, q_t and q_t^*, would cause a technical problem when we solve for steady states. For example, if we consider the following preference,

$$C_t = \left[\gamma_H^{\frac{1}{\eta}}(q_tC_{H,t})^{\frac{\eta-1}{\eta}} + (1-\gamma_H)^{\frac{1}{\eta}}\left((q_t^*)\,C_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

we can not normalized C_t by $q_t^{\frac{\alpha}{1-\alpha}}N_t$ to derive c_t that satisfies the steady state condition.

Without trade costs, we have $P_{F,t} = \epsilon_t P^*_{F,t}$ and $P_{H,t} = \epsilon_t P^*_{H,t}$. Since we have defined price indexed based on local product, we set $P_{H,t} = P^*_{F,t} = 1$, thus $P_{F,t} = \epsilon_t$ and $P^*_{H,t} = \frac{1}{\epsilon_t}$. The exchange rate, $\epsilon_t = \frac{P_{F,t}}{P_{H,t}}$, is also the relative price between foreign product and home product, and the term of trade for the foreign country. P_t and P^*_t becomes price indexes that only depend on two key variables, ϵ_t and Q_t. For example, $P(\epsilon_t) = \left[\gamma_H + (1-\gamma_H)\left(\epsilon_t Q_t^{\theta_H}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$. And the real exchange rate is given by

$$RER_t = \frac{\epsilon_t P^*_t}{P_t} = \frac{\left[\gamma_F\left(\epsilon_t\right)^{1-\eta} + (1-\gamma_F) Q_t^{\theta_F(\eta-1)}\right]^{\frac{1}{1-\eta}}}{\left[\gamma_H + (1-\gamma_H)\left(\epsilon_t Q_t^{\theta_H}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}}. \tag{20}$$

Using the optimal consumption conditions and relative prices, the balance of trade becomes

$$TB_t = (1-\gamma_F)\left(\frac{1}{\epsilon_t P^*_t}\right)^{-\eta} Q_t^{\theta_F(\eta-1)} C^*_t - \epsilon_t\left[(1-\gamma_H)\left(\frac{\epsilon_t}{P_t}\right)^{-\eta}\frac{C_t}{Q_t^{\theta_H(\eta-1)}} + I_{F,t}\right]. \tag{21}$$

In this basic model, we do not allow international lending and borrowing. Therefore, the balance of payments constraint implies $TB_t = 0$, which endogenously determines the exchange rate, ϵ_t.

The goods market of both economies should be clear, thus

$$Y_{H,t} = C_{H,t} + I_{H,t} + C^*_{H,t}, \tag{22}$$

$$Y_{F,t} = C_{F,t} + I_{F,t} + C^*_{F,t} + I^*_{F,t}. \tag{23}$$

Before we proceed, let us introduce the following normalized variables, $k_t = \frac{K_t}{q_t^{\frac{\alpha}{1-\alpha}} N_t}$, $y_t = \frac{Y_t}{q_t^{\frac{\alpha}{1-\alpha}} N_t} = k_t^\alpha$, $c_t = \frac{C_t}{q_t^{\frac{\alpha}{1-\alpha}} N_t}$, $i_{F,t} = \frac{I_{F,t}}{q_t^{\frac{\alpha}{1-\alpha}} N_t}$, $i_{H,t} = \frac{I_{H,t}}{q_t^{\frac{\alpha}{1-\alpha}} N_t}$, which represent per capita capital stock, output, consumption, and investments normalized by product quality index, respectively. Similarly, we normalize foreign variables by $(q^*_t)^{\frac{\alpha}{1-\alpha}} N^*_t$.

Therefore, using optimal consumption allocations and market clear conditions, we have

$$y^*_t = (1-\gamma_H)\left(\frac{P_t}{\epsilon_t}\right)^\eta c_t Q_t^{\frac{\alpha}{1-\alpha}-\theta_H(\eta-1)} + \gamma_F\left(P^*_t\right)^\eta c^*_t + i^*_{F,t} + i_{F,t} Q_t^{\frac{\alpha}{1-\alpha}}, \tag{24}$$

$$y_t = (1-\gamma_F)\left(\epsilon_t P^*_t\right)^\eta c^*_t Q_t^{\theta_F(\eta-1)-\frac{\alpha}{1-\alpha}} + \gamma_H\left(P_t\right)^\eta c_t + i_{H,t}. \tag{25}$$

In addition, the balance of payments constraint is given by

$$i_{F,t} = (1-\gamma_F)\,(P_t^*)^{\eta}\,\epsilon_t^{\eta-1} c_t^* Q_t^{\theta_F(\eta-1)-\frac{\alpha}{1-\alpha}} - (1-\gamma_H)\,(P_t)^{\eta}\,\epsilon_t^{-\eta} c_t Q_t^{-\theta_H(\eta-1)}. \tag{26}$$

3 Economic Equilibrium

We now proceed to derive the macroeconomic equilibria of this two-country system. We start with the foreign country's optimization. And later, we move on to the home country's problem.

3.1 Foreign Country's Problem

For the foreign country, the intertemporal dynamic problem can be treated as a direct implication of the standard Ramsey–Cass–Koopmans growth model, which is described by the following two differential equations:

$$\frac{\dot{c}_t^*}{c_t^*} = \frac{1}{\sigma}\left(r_t^* - \delta - n - \rho - \frac{\alpha}{1-\alpha}\sigma g - \hat{P}_t^*\right), \tag{27}$$

$$\dot{k}_t^* = (k_t^*)^{\alpha} - \left(n + \delta + \frac{\alpha}{1-\alpha} g\right) k_t^* - P_t^* c_t^*. \tag{28}$$

where $\hat{P}_t^* = \frac{\dot{P}_t^*}{P_t^*}$ is the change in price level.

Imposing steady state conditions, $\dot{c}_t^* = \dot{k}_t^* = 0$, we can solve for the steady state values of capital and consumption as follows,

$$k_t^* = \left(\frac{\delta+n+\rho}{\alpha} + \frac{\sigma}{1-\alpha} g - \frac{\hat{P}_t^*}{\alpha}\right)^{\frac{1}{\alpha-1}}, \tag{29}$$

$$c_t^* = \frac{1}{P_t^*}\left[(k_t^*)^{\alpha} - \left(n + \delta + \frac{\alpha}{1-\alpha} g\right) k_t^*\right]. \tag{30}$$

In the absence of price change, $\hat{P}_t^* = 0$, this set of solution is consistent with the steady state solution of a standard Ramsey–Cass–Koopmans model, such that $k_t^* = \bar{\kappa}$

and $c_t^* P_t^* = \bar{\zeta}$, where

$$\bar{\kappa} \equiv \left(\frac{\delta + n + \rho}{\alpha} + \frac{\sigma}{1-\alpha} g \right)^{\frac{1}{\alpha - 1}}, \tag{31}$$

$$\bar{\zeta} \equiv \left[\bar{\kappa}^{\alpha} - (n + \delta + \frac{\alpha}{1-\alpha} g) \bar{\kappa} \right]. \tag{32}$$

3.2 Home Country's Problem

Because of the quality improvement mechanism, the home country has a complicated optimization problem. For an economic agent in home country, denoted by superscript i, the consumption per capita and capital stock per capita are given by $C_t^i = \frac{C_t}{N_t}$, $K_t^i = \frac{K_t}{N_t}$. In addition, as we assume that the representative firms rent capital and employ labor from the household, it is the agent in the representative household that chooses consumption and investment composition to maximize the following life-time utility

$$\max \int_0^{\infty} \frac{(C_t^i)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt,$$

which is subject to the following constraints[7]

$$\begin{aligned} R_t q_t K_t^i + W_t &= P_t C_t^i + P_{H,t} I_{H,t}^i + P_{F,t} \phi_t K_t^i, \\ \dot{K}_t^i &= I_{H,t}^i + (\phi_t - \delta - n) K_t^i, \\ \dot{q}_t &= \left(Q_t g + \frac{1 - Q_t}{Q_t} \phi_t \right) q_t. \end{aligned}$$

The maximum principle can be used to handle such a problem.[8] We define a standard Hamiltonian, where μ_t and ν_t are called the costate variable, and λ_t is a Lagrange multiplier,

$$\begin{aligned} \mathscr{H}\left(C_t^i,\ K_t^i,\ q_t,\ \phi_t,\ I_{H,t}^i,\ t\right) = & \frac{(C_t^i)^{1-\sigma} - 1}{1-\sigma} + \lambda_t \left(R_t q_t K_t^i + W_t - P_t C_t^i - P_{H,t} I_{H,t}^i - P_{F,t} \phi_t K_t^i \right) \\ & + \mu_t \left(I_{H,t}^i + (\phi_t - \delta - n) K_t^i \right) + \nu_t \left[\left(Q_t g + \frac{1 - Q_t}{Q_t} \phi_t \right) q_t \right]. \end{aligned}$$

[7]The share of capital stock owned by agent i, K_t^i, has a quality measure, q_t^i. Sine all economic agent are identical, $q_t^i = q_t$. The law of motion for q_t^i can be replaced by equation (6).

[8]Obstfeld and Rogoff (1996) discussed a general procedure to solve such a maximization problem on page 748.

The first-order conditions (FOC) are given by

$$\frac{\partial \mathscr{H}}{\partial C_t^i} = \left(C_t^i\right)^{-\sigma} - \lambda_t P_t, \tag{33}$$

$$\rho - \frac{\dot{\mu}_t}{\mu_t} = \frac{\lambda_t}{\mu_t}\left(R_t q_t - P_{F,t}\phi_t\right) + (\phi_t - \delta - n), \tag{34}$$

$$\rho - \frac{\dot{\nu}_t}{\nu_t} = \frac{\lambda_t}{\nu_t} R_t K_t^i + 2Q_t g - \phi_t, \tag{35}$$

$$\frac{\partial \mathscr{H}}{\partial \phi_t} = -\lambda_t P_{F,t} K_t^i + \mu_t K_t^i + \nu_t \frac{1 - Q_t}{Q_t} q_t, \tag{36}$$

$$\frac{\partial \mathscr{H}}{\partial I_{H,t}^i} = \mu_t - \lambda_t P_{H,t}, \tag{37}$$

where $P_{H,t} = 1$, $P_{F,t} = \epsilon_t$, and $P_t = \left[\gamma_H + (1-\gamma_H)\left(\epsilon_t Q_t^{\theta_H}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$.

Equation (36) provides the first-order condition for ϕ_t. The first term represents the cost of using the foreign capital goods, the second term stands for the cost of using domestic capital goods, while the last term measures the quality benefit that comes from capital import. In addition, equation (37) implies that $\mu_t = \lambda_t$. Therefore, we can rewrite equation (36) as $\frac{\partial \mathscr{H}}{\partial \phi_t} = \lambda_t K_t^i (1 - \epsilon_t) + \nu_t \frac{1-Q_t}{Q_t} q_t$, where $\lambda_t > 0$ and $\nu_t \geqslant 0$. Thus, the optimal choice of ϕ_t depends on the exchange rate. In addition, because of the linearity of the Hamiltonian function with respect to the variable ϕ_t, it has a bang-bang solution. If $\epsilon_t < 1$, we have $\frac{\partial \mathscr{H}}{\partial \phi_t} > 0$, as a result, the optimal value for ϕ_t should take the maximum value that is available; if $\epsilon_t = 1$, $\frac{\partial \mathscr{H}}{\partial \phi_t} = 0$ implies $\nu_t = 0$ or $Q_t = 1$; if $\epsilon_t > 1$ and $Q_t = 1$, $\frac{\partial \mathscr{H}}{\partial \phi_t} < 0$, thus $\phi_t = 0$; and if $\epsilon_t > 1$ and $Q_t < 1$, $\nu_t > 0$. This interaction between quality upgrading and exchange rate will be discussed in section 4.3.

The Euler equation and the law of motion for capital of the home country are given by

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\left[\alpha k_t^{\alpha-1} - \left((\epsilon_t - 1)\phi_t + \delta + n + \rho + \hat{P}_t\right)\right] - \frac{\alpha}{1-\alpha}\hat{q}_t, \tag{38}$$

$$\dot{k}_t = k_t^{\alpha} - \left[n + \delta + \frac{\alpha}{1-\alpha}\hat{q}_t + (\epsilon_t - 1)\,\phi_t\right] k_t - P_t c_t. \tag{39}$$

4 Balanced Growth Paths

We define a balanced growth path as being one along which key economic variables in these two economies can grow at a constant rate. Equations (5) and (6) provide a necessary condition for the balanced growth path in this two-country system, such that

$$\hat{q}_t^* = g = \hat{q}_t = Q_t g + \phi_t(\frac{1}{Q_t} - 1), \tag{40}$$

which yields two solutions, $Q_t = 1$, or $Q_t < 1$ and $\phi_t = Q_t g$. As a result, in our model, there are two potential balanced growth paths.

Proposition 1. *Balanced Growth Paths. For $Q_t = \bar{Q} = 1$, or $\exists\ \tilde{Q} \in (0,\ 1)$, $Q_t = \tilde{Q}$, output per capita, consumption per capita, and capital per capita in the two-country system can grow along the balanced growth path at the constant rate, $\frac{\alpha}{1-\alpha}g$.*

4.1 Balanced Growth Path with $\bar{Q} = 1$

For $Q_t = \bar{Q} = 1$, we have $q_t = q_t^*$, which implies that both countries share the same quality level. According to our discussion of equation (36), there are three distinct scenarios that depend on the equilibrium exchange rate ϵ_t, which are summarized by the following proposition.

Proposition 2. *Balanced Growth Path with $\bar{Q} = 1$. For $Q_t = \bar{Q} = 1$, the balanced of payments constraint determines equilibrium exchange rate, $\bar{\epsilon}$, which characterizes three equilibria with balanced growth path. And all major variables of these two economies can grow at a constant rate, $\frac{\alpha}{1-\alpha}g$.*

1. *If $\bar{\epsilon} = 1$, we have $P_t = P_t^* = 1$. Thus $k_t = k_t^* = \bar{\kappa}$, $c_t = c_t^* = \bar{\zeta}$. And $\bar{\phi} = (\gamma_H - \gamma_F)\frac{\bar{\zeta}}{\bar{\kappa}} \geqslant 0$. In particular, $\bar{\phi} = 0$ if and only if $\gamma_H = \gamma_F$.*

2. *If $\bar{\epsilon} < 1$, we have $P_t < 1 < P_t^*$, $k_t \geqslant \bar{\kappa} = k_t^*$ and $c_t > \bar{\zeta} > c_t^*$. And $\bar{\phi} = \max\left\{0, \frac{i_{F,t}}{k_t}\right\}$, where*

$$i_{F,t} = (1-\gamma_F)\left(P_t^*\right)^\eta \bar{\epsilon}^{\eta-1} c_t^* - (1-\gamma_H)\left(P_t\right)^\eta \bar{\epsilon}^{-\eta} c_t.$$

3. *If $\bar{\epsilon} > 1$, we have $P_t > 1 > P_t^*$, $k_t = \bar{\kappa} = k_t^*$, $c_t < \bar{\zeta} < c_t^*$, and $\bar{\phi} = 0$.*

Although importing foreign capital goods does not affect the relative quality at $Q_t = \bar{Q} = 1$, the choice of importing foreign capital goods to the home country still depends on the following balance of trade condition,

$$(1-\gamma_F)\left(P_t^*\right)^\eta \epsilon_t^{\eta-1} c_t^* - (1-\gamma_H)\left(P_t\right)^\eta \epsilon_t^{-\eta} c_t = 0, \tag{41}$$

which directly comes from the trade balance equation with $\phi_t = 0$. If the solution of equation (41) for ϵ_t provides that $\bar{\epsilon} > 1$, the $\bar{Q}$ steady state is a single point $\{Q = 1,\ \epsilon_t = \bar{\epsilon}\}$; if $\bar{\epsilon} \leqslant 1$, the $\bar{Q}$ steady state is a set $\{Q = 1,\ \bar{\epsilon} \leqslant \epsilon_t \leqslant 1\}$, and every point that belongs to this set is a steady state, where $\bar{\phi}$ will be chosen to satisfies the trade balance condition, thus $\dot{\epsilon}_t = 0$.

The $\bar{Q}$ steady state is a simple extension of the standard Ramsey model. The home country and the foreign country are almost identical, and follow the same balanced growth path to grow at rate $\frac{\alpha}{1-\alpha}g$. If this $Q = 1$ steady state is reached, we argue that the home economy have caught up with the foreign economy.

4.2 Balanced Growth Path with $\tilde{Q} < 1$

Let us now return to the case that the home country is a less developed country with $q_0 < q_0^*$. For any $Q_t \in (0,1)$, equation (6) implies that there exists $\tilde{\phi}(Q_t)$ that satisfies $\hat{q} = g$,

$$\tilde{\phi}(Q_t) = Q_t g. \tag{42}$$

This provides a necessary and sufficient condition for the home country to maintain a constant relative quality with the foreign country. Therefore, in this case, importing high-quality capital goods plays a key role in enabling the home country to keep up with the foreign country. And the foreign country will always be the leading economy in this two-country system.

Proposition 3. *Balanced Growth Path with $\tilde{Q} \in (0,\ 1)$ exists if and only if the solution to the following equations of Q_t and ϵ_t satisfies $0 < Q_t < 1$ and $\epsilon_t > 1$,*

$$(1-\gamma_F)\left(P_t^*\right)^\eta \epsilon_t^{\eta-1} c_t^* Q_t^{\theta_F(\eta-1)-\frac{\alpha}{1-\alpha}} - \frac{1-\gamma_H}{Q_t^{\theta_H(\eta-1)}}\left(P_t\right)^\eta \epsilon_t^{-\eta} c_t - Q_t g k_t = 0, \tag{43}$$

$$\frac{n+\delta+\rho+\frac{\alpha}{1-\alpha}\sigma g}{\epsilon_t - 1}\frac{1-Q_t}{Q_t} - \rho - \frac{\alpha}{1-\alpha}\sigma g + \frac{\alpha}{1-\alpha} g = 0, \tag{44}$$

where $k_t = \left(\frac{\delta+n+\rho+(\epsilon_t-1)Q_t g}{\alpha} + \frac{\sigma}{1-\alpha}g\right)^{\frac{1}{\alpha-1}} < \bar{\kappa}.$

The first condition in proposition 3 is derived from the balance of payments constraint for quality improvement, which establishes the boundary that the home country is able to maintain a constant relative quality index with the foreign country. This is called the balance of payments constraint for quality improvement locus, or simply BOP locus. For any given ϵ, if Q is above this locus, the balance of trade constraint implies that $i_{F,t} > \tilde{\phi}(Q_t)k_t$, thus $\dot{Q}_t > 0$, meaning that the balance of payments constraint is unbounded for quality improvement and growth. This region is marked as "feasible". If Q is below the BOP locus, the balance of payments constraint implies that $i_{F,t} < \tilde{\phi}(Q_t)k_t$, thus, $\dot{Q}_t < 0$, the home country diverges from the foreign country. This information is summarized in Figure 1, with arrows that illustrate the direction of motion for Q.

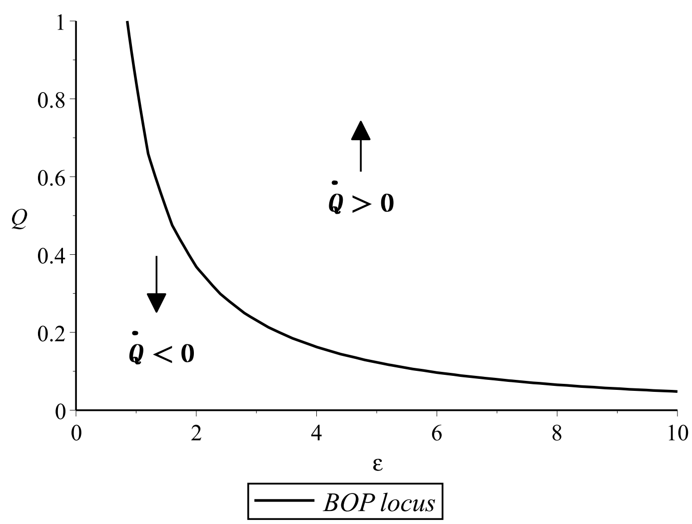


Figure 1: The balance of payments constraint for quality improvement (BOP)

The second condition in proposition 3 is derived from the first-order condition of optimal capital import with $\phi_t = Q_t g$ as the optimal solution, which establishes a direct linkage between Q_t and ϵ_t. Thus, for $\epsilon_t > 1$, we have

$$Q_t(\epsilon_t) = \frac{n+\delta+\rho+\frac{\alpha}{1-\alpha}\sigma g}{(\epsilon_t-1)(1+\rho+\frac{\alpha}{1-\alpha}\sigma g - \frac{\alpha}{1-\alpha}g) + n+\delta+\rho+\frac{\alpha}{1-\alpha}\sigma g}, \tag{45}$$

which describes the locus of optimal capital import (OCI) for quality improvement that satisfies $\dot{Q} = 0$. For points to the right of this curve, the foreign capital goods are too expensive to import for investment,[9] thus, the optimal capital import share,ϕ_t, is less than $\tilde{\phi}$, thus, $\dot{Q} < 0$. For points to the left of this OCI locus, the quality benefit that comes from importing foreign capital goods overtakes the high foreign price, $\phi_t > \tilde{\phi}$, thus $\dot{Q} > 0$. And the latter region is marked as "optimal" for quality improvement, meaning that domestic households prefer to import capital goods to improve Q. The directions of movement for Q are displayed on the $Q - \epsilon$ plane by Figure 2.

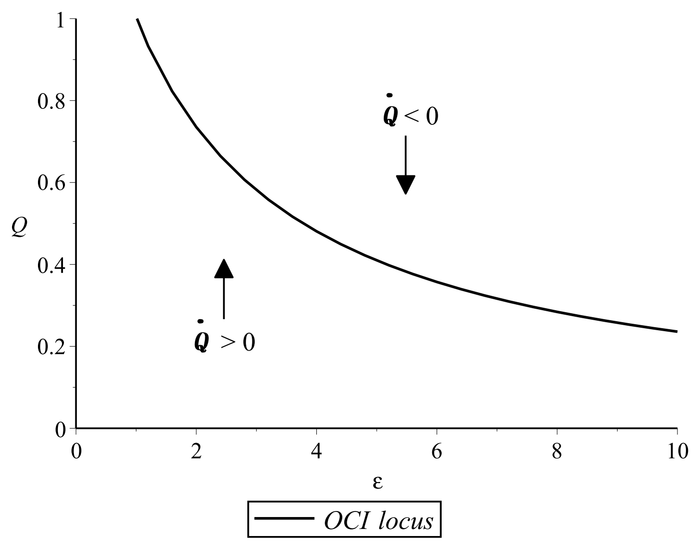


Figure 2: The optimal capital import (OCI) locus

The interaction of the BOP locus and the OCI locus determines the dynamic features of Q and ϵ on the $Q-\epsilon$ plane, as being summarized in Table 1. In particular, the steady state of $\tilde{Q}$ is determined by the intersection of these two curves.

- If an economy only satisfies the feasibility condition, the representative household is feasible to choose a large sufficient level of capital import to improve Q, but chooses to not do so (not optimal), $\dot{Q} < 0$. As a result, a relatively low capital import causes the exchange rate to appreciate, $\dot{\epsilon} < 0$.

- If an economy only satisfies the optimality condition, the representative household intend to choose a high capital import level that can not satisfies the balance of payments constraint with current exchange rate. Since international borrowing is not allowed, this state is unstable, the exchange rate has to sharply depreciate to satisfies the feasibility condition.

[9] Recall that the first-order condition for ϕ_t, equation (36), for a larger ϵ_t, ϕ_t becomes smaller.

- If neither of these two conditions can be satisfied, we will have $\dot{Q} < 0$. When this low capital import choice is still bounded by the balance of payments constraint, we will have $\dot{\epsilon} > 0$, otherwise, $\dot{\epsilon} \leqslant 0$.

- If both feasibility and optimality conditions can be satisfied, we have $\dot{Q} > 0$. However, the change of exchange rate is still determined by whether the balance of payments constraint is bounded.

Table 1: The interaction of balance of payments constraint and optimal capital import

BOP locus	OCI locus		
	Above	On	Beneath
Above	Feasible, Not Optimal $\dot{Q} < 0$, $\dot{\epsilon} < 0$	Feasible, - $\dot{Q} = 0$, $\dot{\epsilon} < 0$	Feasible, Optimal $\dot{Q} > 0$, $\dot{\epsilon}$ uncertain
On	-, Not Optimal $\dot{Q} < 0$, $\dot{\epsilon} < 0$	-, - $\tilde{Q}$ steady state	-, Optimal Unstable, $\dot{\epsilon} > 0$
Beneath	Not Feasible, Not Optimal $\dot{Q} < 0$, $\dot{\epsilon}$ uncertain	Not Feasible, - Unstable, $\dot{\epsilon} > 0$	Not Feasible, Optimal Unstable, $\dot{\epsilon} > 0$

4.3 The Dynamics of Q

Following a standard phase diagram analysis, we can evaluate the existence of these two types of steady states and characterize their dynamic features,

In our model, the steady state with $\tilde{Q}$ is determined by the intersection of the BOP locus and the OCI locus, while the steady state with $Q = 1$ is determined by the intersection of trade balance condition at $Q = 1$, equation (41).[10]

For the sake of simplicity, we assume a minimum foreign demand structure for consumption goods produced in the home country, and replace equation (14) by

$$c^*_{H,t} = \Gamma \left(P^*_{H,t}\right)^{-\eta} Q_t^{\theta_F(\eta-1)}, \tag{46}$$

where $\Gamma = (1-\gamma_F)\left(P^*_t\right)^{\eta} c^*_t$ and we assume $P^*_t = 1$. Therefore, the two-country model is reduced to a small open economy model. We allow the foreign country to grow at

[10]Comparing equations (41) and (43), we know that the solution $(\bar{\epsilon})$ that satisfies the trade balance condition at $Q = 1$ is to the left of the BOP locus.

a steady state, where $c^* = \bar{\zeta}$ and $k^* = \bar{\kappa}$. Since c^* is given as a constant, Γ is mainly affected by parameter γ_F, the home bias parameter in the foreign country.

Table 2: Common parameters

Parameter	δ	g	n	α	γ_H	ρ	σ	η
Value	0.03	0.02	0.01	0.4	0.8	0.03	1	2

Table 3: Case-specific parameters

Case	Parameter		
	θ_H	θ_F	γ_F
Figure 3 (a)	3	3	0.95
Figure 3 (b)	2	2	0.5
Figure 4 (a)	1	1	0.8
Figure 4 (b)	1	5	0.5

The following figures are simulated using various sets of parameters. Table 2 summarizes the parameter values that are commonly used across models, including capital depreciation rate δ, foreign quality improvement rate g, population growth rate n, production function parameter α, home country consumption weight γ_H, preference parameter ρ and σ, and the trade elasticity η. And Table 3 lists the case-specific parameters, such as quality adjustment parameters θ_H and θ_F, and foreign demand parameter Γ.[11] One feature worth noting is that these case-specific parameters do not enter equation (45), meaning that the OCI locus is unaffected.

According to our proposition 1, $Q = 1$ is always a steady state, which is marked by letter D, whereas the existence of steady state with $\tilde{Q} \in (0,\ 1)$, marked by E, is uncertain. Therefore, we discuss the following two cases, which are constructed based on the number of intersections of the OCI locus and the BOP locus.

4.3.1 Zero $\tilde{Q}$ Equilibrium

When the BOP locus and OCI locus have zero intersections in the interval of $(0,\ 1)$ for Q, the steady state with $\tilde{Q}$ doesn't exist. Figure 3 displays two possible scenarios, where the arrows show the directions of movement for both ϵ and Q. On the $\epsilon - Q$

[11] We do not stress the economic intuitions in these exercises, since we would like to cover most scenarios that are theoretically possible.

plane, these two curves can be roughly parallel, or they can turn to intersect at $Q > 1$. The BOP locus can be either above or beneath the OCI curve.

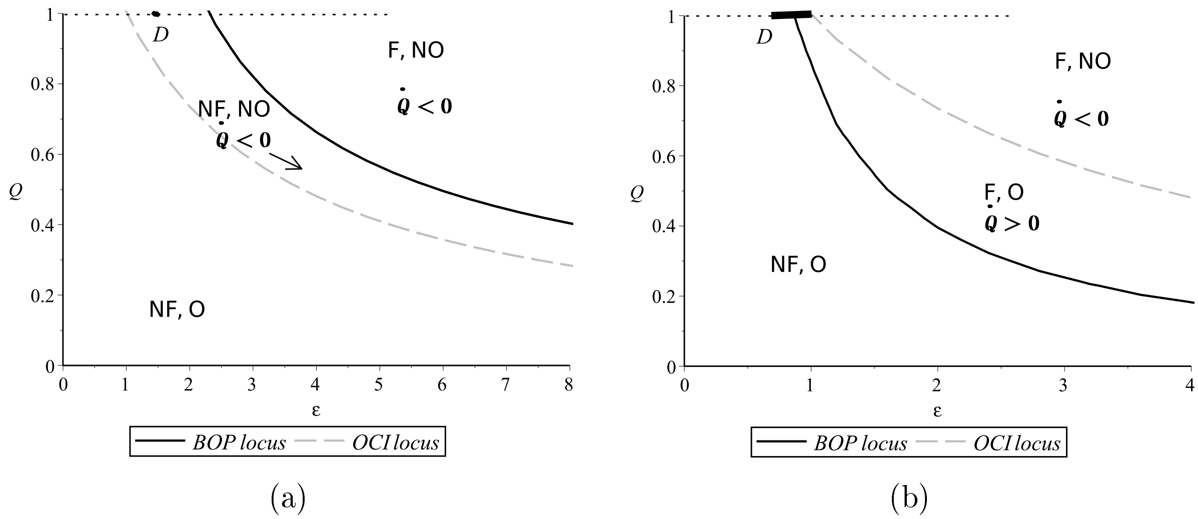


Figure 3: Zero $\tilde{Q}$ equilibrium

Note: F, NF, O, NO stand for "Feasible", "Not Feasible", "Optimal", and "Not Optimal" respectively. The steady state $\bar{Q}$ in panel (a) is given by $\bar{\epsilon} = 1.57$, while the steady state $\bar{Q}$ in panel (b) is given by $0.79 < \epsilon_t \leqslant 1$.

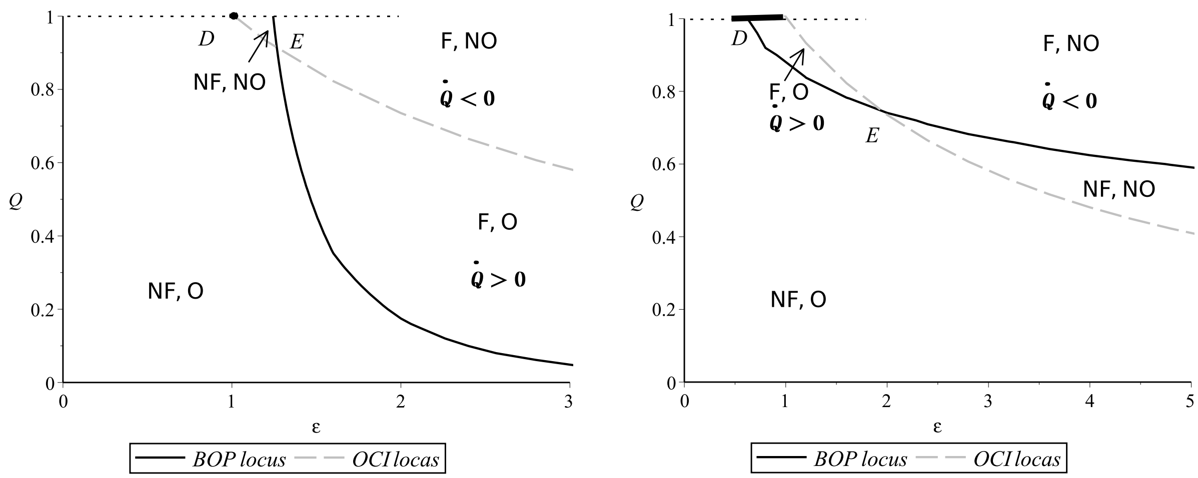
Figure 3 panel (a) provides an example in which the BOP locus is above the OCI locus. It shows that no points on the $\epsilon - Q$ plane could simultaneously satisfy the two conditions in proposition 3. For any initial point, there is no dynamic path for the home economy to catch-up with the technology frontier.

Panel (b) of Figure 3 illustrates a different case in which the BOP locus is beneath the OCI locus. And steady state D (a set of steady states with $Q = 1$) appears to be stable. There exists an optimal growth path lying between the BOP locus and the OCI locus and passing a point that belongs to set D. Thus, panel (b) demonstrates a region wherein quality upgrading and convergence would take place.

4.3.2 The $\tilde{Q}$ Equilibrium

When the BOP locus intersects with the OCI locus only once within the interval of $(0,\ 1)$ for Q, we have one unique $\tilde{Q}$ steady state, denoted by E. Figure 4 demonstrates two examples.

Panel (a) of Figure 4 illustrates the scenario wherein the BOP locus crosses the OCI locus at point E from above. The arrows around point E indicate that this steady state



(a) (b)

Figure 4: One $\tilde{Q}$ steady state
Note: F, NF, O, NO stand for "Feasible", "Not Feasible", "Optimal", and "Not Optimal" respectively. The steady state $\bar{Q}$ in panel (a) is given by $\bar{\epsilon} = 1$, while the steady state $\bar{Q}$ in panel (b) is given by $0.59 < \epsilon_t \leqslant 1$.

is locally stable. For an economy with initial quality index $Q_0 < \tilde{Q}$, there is a dynamic path that leads to point E. However, the equilibrium point D, in this case, is unstable. Thus, point E is a dominant steady state. Since $\tilde{Q} < 1$, the quality upgrade in the home country will prematurely stop at $\tilde{Q}$. Comparing this result with the example of Figure 3 panel (b), we find that this is mainly caused by the relatively high home bias in the consumption demand of the foreign country. This feature is consistent with the empirical finding of Henn et al. (2013) that the quality convergence is particularly rapid during the early stages of development and completes as a country reaches upper middle-income status.

Figure 4 panel (b) depicts the scenario wherein the BOP locus crosses the OCI locus from another direction. The intersection point E is a $\tilde{Q}$ steady state, but it is unstable. According to Table 3, one key difference arises: the quality punishment factor θ_F is large, thus the foreign demand drops rapidly with lower product quality. As a result, this equilibrium generates two contrary dynamic paths: for an economy with initial product quality that is relatively close to the foreign leader ($Q_0 > \tilde{Q}$), it could converge to the set of steady state D, reaching $Q = 1$; for a poorer economy with low initial product quality ($Q_0 < \tilde{Q}$), in contrast, it is severely constrained by the balance of payment. This country cannot simultaneously satisfy the feasibility and optimality conditions, and has to deteriorate to the lower right corner of this $\epsilon - Q$ plane. Therefore, in this case, both convergence and divergence of income would take place.

5 Discussion

In this section, we revisit a series of stylized facts and apply our model to shed light on the underlying relationship between trade and economic growth.

5.1 Import Share of Investment

An important piece of empirical evidence that reveals the role of capital goods import involves a trade-off between consumption and investment in a country's imports. Based on data from the World Input-Output Database (Timmer, 2012), Table 4 lists the percentage of imports that is used for investment for ten countries during 1996 and 2009, and shows that rapid developing economies spend larger shares of import on investment. For example, the investment shares of import could be higher than 60 percent in China, India, Korea, and Turkey, while they can be lower than 30 percent in developed countries like the United Kingdom and Japan.

Table 4: Investment share (% of imported final expenditure)

	1996	2000	2004	2009
China	66.6	68.2	65.6	55.4
France	30.5	32.0	29.6	32.7
India	48.0	35.3	65.5	64.5
Japan	27.9	32.5	31.5	27.8
Korea	61.7	52.3	50.0	44.8
Mexico	44.9	49.0	44.5	39.7
Taiwan	40.0	49.0	45.7	40.0
Turkey	69.1	58.7	56.8	50.4
U.K.	34.8	31.5	21.9	19.2
U.S.	43.5	44.3	39.5	36.0

Source: Author's calculation using World Input-Output Database, 1996-2009.

Since the United States is often referred as the world economic leader, we use the PPP converted GDP per capita relative to the United States as an approximate indicator for Q (Penn World Table version 7.1). Figure 5 illustrates a negative relationship between capital goods import (for investment) and relative income to the United States.

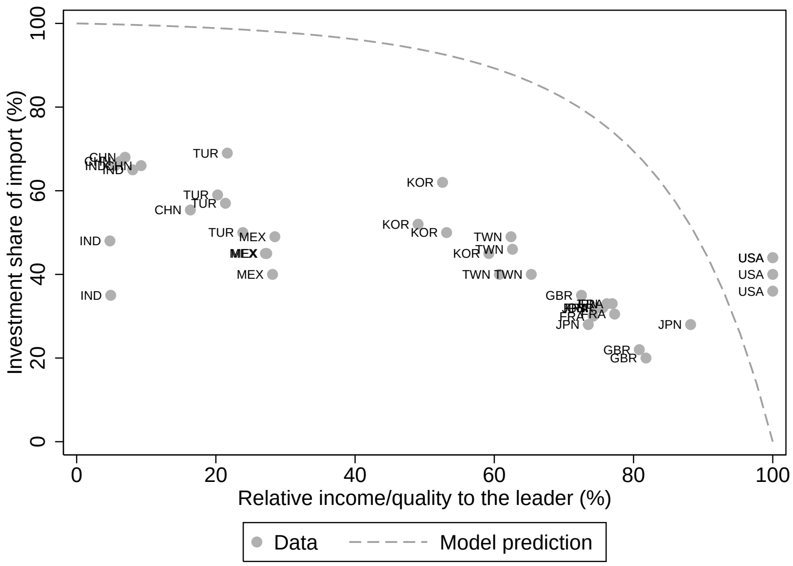


Figure 5: Investment share of import (data vs model prediction)

Note: Parameter values are given by Table 2 and the case of Figure 4 panel (a).

This result is qualitatively consistent with the implication of our model. The dash line in Figure 5 depicts the capital goods import shares along the OCI locus, which continuously decreases as Q increases. When Q is lower, the benefit on quality improvement from importing foreign capital goods is relatively large. As a result, individuals would spend a larger portion of international trade revenue on high-quality foreign capital inputs. As Q approaches to 1, importing foreign capital goods become less attractive, thus the investment share drops and is gradually replaced by household consumption.

5.2 Trade Balance and Exchange Rate Reversal

One of the prevalent illusions of Asian economic growth is that the rapid economic expansions are associated with chronic trade surpluses. This phenomenon has been heavily criticized as a "beggar-thy-neighbor" policy, which involves exporting unemployment to other nations and influencing foreign labor market structures. **?** shows that U.S. trade deficits could account for about 30 percent of the overall employment share decrease in American manufacturing. Bernanke (2005) further argued that it is a primary cause of the global current account imbalances.

The economic history tells a different story: major Asian economies did not develop large trade surpluses in the early stage of development until they passed a certain threshold, at which point their trade balances turned into surpluses. This trade balance reversion can take several years to complete, during which the trade balances fluctuate up and down between deficits and surpluses. This took place in Japan and Taiwan between 1965 to 1980, in South Korea between 1977 to 1996, and in China during the 1990s.

This trade balance reversal is associated with the dynamics of real exchange rate. Following Rodrik (2008), we use data from Penn World Tables 7.1 (Heston, Summers, and Atina 2012) to calculate a "real" exchange rate (RER)

$$\ln RER_{it} = \ln\left(\frac{XRAT_{it}}{PPP_{it}}\right),$$

where i is an index for countries and t is an index for time periods. Exchange rates ($XRAT$) and PPP conversion factors (PPP) are expressed as national currency units per U.S. dollar. Figure 6 depicts the change of the real exchange rate of five economies during 1950-2010: China, Japan, South Korea, Singapore, and Taiwan. It shows that China, South Korea, and Taiwan shared a similar relationship between growth and real exchange rate adjustment, which first depreciated, and then reversed to appreciate. In developing countries, real exchange rates significantly affect economic growth, as overvaluation hurts growth while undervaluation facilitates it (Rodrik, 2008). This relationship is stronger for developing countries, but disappears for advanced countries. Thus, it suggests that the real exchange rate is associated with some fundamental factors in the process of economic convergence.

Government Debt: an Extension of the Baseline Model

Since we assume no foreign capital flows in the basic model, the area that is "Not Feasible, but Optimal" is actually inaccessible: the exchange rate has to depreciate sharply to ensure the trade balance is zero. As a result, in the example that is illustrated by Figure 4 panel (b), a low income country is unable to catch up with the foreign country. Therefore, it would be interesting to investigate whether foreign aid is able to help.

We assume that the government in the home country is able to borrow in the capital market to finance trade deficits, as long as the government debt is below the debt ceiling,

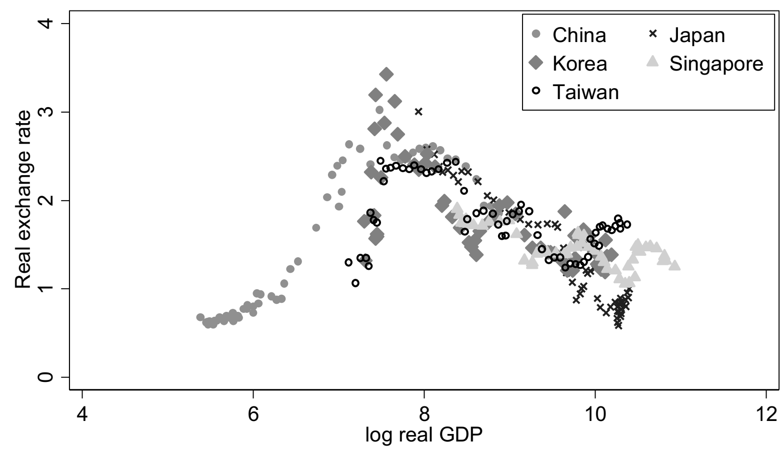


Figure 6: Real exchange rate dynamics

$B_t \leqslant \bar{B}$, such that

$$TB_t = B_{t+1} - R_{B,t}B_t,$$

where $R_{B,t}$ represents the interest rate of government bond. In addition, we assume the adjustment of exchange rate is sluggish and depends on a function of trade balance, $\tau(\cdot)$

$$\frac{\dot{\epsilon}}{\epsilon} = \tau\left(tb_t\right), \tag{47}$$

where $tb_t = (1-\gamma_F)\left(P_t^*\right)^{\eta} \epsilon_t^{\eta-1} c_t^* Q_t^{\theta_F(\eta-1)-\frac{\alpha}{1-\alpha}} - (1-\gamma_H)\left(P_t\right)^{\eta} \epsilon_t^{-\eta} c_t Q_t^{-\theta_H(\eta-1)} - i_{F,t}$.

Following these two assumptions, the home economy that starts with an initial state that is considered to be "Not Feasible, but Optimal" can catch up with the foreign economy, while the exchange rate depreciates and foreign debts accumulate over time.

Figure 7 illustrates the example of Figure 4 panel (b) with international borrowing. It shows that if the government can borrow to finance the capital good import, a growth strategy emerges: the home country can take a saddle path (dash arrow line) to reach equilibrium point E. However, this stage of growth heavily relies on accumulating foreign debt to finance persistent trade deficits, which makes the home economy more vulnerable to external shocks and currency crisis. After passing point E, this economy becomes self-reliance and has the option to reach the set of steady state D. An overview of the whole process indicates that two patterns would reverse before and after passing point E: the trade balances move from deficit to surplus and the exchange rate turns from depreciation to appreciation.

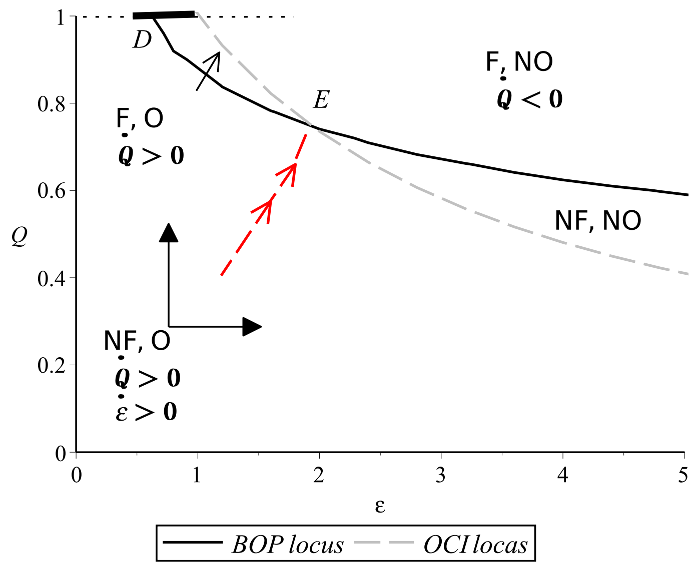


Figure 7: Government debt and quality improvement

6 Concluding Remarks

We have developed a growth model that features quality upgrading as the primary driver of income convergence. As we assume that technology is embodied in high-quality capital goods, the model emphasizes the import of foreign capital goods as the main channel of international technology diffusion, which could be restricted by the balance of payments constraint. We characterize the possible steady states and discuss their dynamic features.

Our model consolidates existing empirical evidence on the East Asian growth experience. Importing foreign capital goods improves the quality of domestic capital stock and output through an intensive factor accumulation process, and consequently promotes industry upgrading (measured by quality improvement). Since import expansion is subject to the balance of payments constraint, persistent export expansion is needed to finance rising capital goods import, while better product quality ensures growing foreign demand. It suggests that the factor accumulation, industry upgrade, and export orientation strategy can be considered as three by-products of economic development.

The role of product quality is in keeping with the empirical findings by Hallak (2006) and Feenstra and Romalis (2014) that rich countries import more and consume more from countries producing high-quality goods. Thus, a potential barrier of growth is the low product quality in developing countries. Our analysis shows that for a developing economy with very poor product quality, it would be hurt badly by the balance of payments constraint, since the foreign demand is very limited. It also implies that countries that are close to the economic leader have better opportunities to catch-up to the frontier.

Our framework yields predictions about the dynamics of trade balance and exchange rate. The phase diagram analysis clearly indicates the scenario with trade balance reversal and exchange rate reversal, which is consistent with the empirical observations in a few Asian economies. One thing worth noting is that the exchange rate, generally speaking, appreciates along with quality improvements. Thus, without non-tradable goods, our model yields the dynamic pattern for the exchange rate that is very similar to the Balassa-Samuelson effect.

Finally, our framework is simple enough to allow for extensions and variations. For example, it is straightforward enough to allow multiple sectors in our model to analyze structural change. Then the model could quantitatively evaluate the economic growth and structural transformation for a specific economy. We will pursue such an extension in our ongoing research.

Appendix

Proofs of Propositions

Proposition. *2 Balanced Growth Path with $\bar{Q} = 1$. For $Q_t = \bar{Q} = 1$, the balanced of payments constraint determines equilibrium exchange rate, $\bar{\epsilon}$, which characterizes three equilibria with balanced growth path. And all major variables of these two economies can grow at a constant rate, $\frac{\alpha}{1-\alpha} g$.*

1. *If $\bar{\epsilon} = 1$, we have $P_t = P_t^* = 1$. Thus $k_t = k_t^* = \bar{\kappa}$, $c_t = c_t^* = \bar{\zeta}$. And $\bar{\phi} = (\gamma_H - \gamma_F) \frac{\bar{\zeta}}{\bar{\kappa}} \geqslant 0$. In particular, $\bar{\phi} = 0$ if and only if $\gamma_H = \gamma_F$.*

2. *If $\bar{\epsilon} < 1$, we have $P_t < 1 < P_t^*$, $k_t \geqslant \bar{\kappa} = k_t^*$ and $c_t > \bar{\zeta} > c_t^*$. And $\bar{\phi} = \max \left\{ 0, \frac{i_{F,t}}{k_t} \right\}$, where*

$$i_{F,t} = (1 - \gamma_F) (P_t^*)^\eta \bar{\epsilon}^{\eta - 1} c_t^* - (1 - \gamma_H) (P_t)^\eta \bar{\epsilon}^{-\eta} c_t.$$

3. *If $\bar{\epsilon} > 1$, we have $P_t > 1 > P_t^*$ $k_t = \bar{\kappa} = k_t^*$, $c_t < \bar{\zeta} < c_t^*$, and $\bar{\phi} = 0$.*

Proof. Let's go over the three scenarios in turn. From equation (26), we have

$$\epsilon_t^{2\eta-1} = \frac{(1-\gamma_H)(P_t)^{\eta-1} c_t}{(1-\gamma_F)(P_t^*)^{\eta-1} c_t^*},$$

which implies that the equilibrium exchange rate, $\bar{\epsilon}$, is determined by preference parameter γ_H , γ_F , elasticity of trade η, and relative consumption.

1. $\bar{\epsilon} = 1$. Since $P_{F,t} = \epsilon_t$ and $P_{H,t}^* = \frac{1}{\epsilon_t}$, we have $P_t = P_t^* = 1$. Equations (5) and (6) are consistent with standard Ramsey–Cass–Koopmans model. Under these specific configuration, we have

$$\begin{aligned} k_t &= k_t^* = \bar{\kappa}, \\ c_t &= c_t^* = \bar{\zeta}. \end{aligned}$$

   According to the balanced trade condition, we have

$$\phi_t = (\gamma_H - \gamma_F)\frac{\bar{\zeta}}{\bar{\kappa}} \geqslant 0.$$

   In particular, $\phi_t = 0$ if and only if $\gamma_H = \gamma_F$.

2. $\bar{\epsilon} < 1$. $P_t < 1 < P_t^*$.

$$\begin{aligned} k_t &= \left(\frac{\delta+n+\rho}{\alpha} + \frac{\sigma}{1-\alpha}g - (1-\bar{\epsilon})\phi_t\right)^{\frac{1}{\alpha-1}} \geqslant \bar{\kappa} = k_t^*, \\ c_t &= \frac{1}{P_t}\left\{k_t^\alpha - \left[n+\delta+\frac{\alpha}{1-\alpha}g + (\bar{\epsilon}-1)\phi_t\right]k_t\right\} \\ &> \frac{1}{P_t}\left[\bar{\kappa}^\alpha - (n+\delta+\frac{\alpha}{1-\alpha}g)\bar{\kappa}\right] > \bar{\zeta} > \frac{\bar{\zeta}}{P_t^*} = c_t^* \end{aligned}$$

   From equations (36) and (26), we have

$$k_t\phi_t = (1-\gamma_F)(P_t^*)^\eta \bar{\epsilon}^{\eta-1}c_t^* - (1-\gamma_H)(P_t)^\eta \bar{\epsilon}^{-\eta}c_t.$$

3. $\bar{\epsilon} > 1$. $P_t > 1 > P_t^*$. Since $\frac{\partial \mathscr{H}}{\partial \phi_t} < 0$, we have$\phi_t = 0$. Thus,

$$\begin{aligned} k_t &= \left(\frac{\delta + n + \rho}{\alpha} + \frac{\sigma}{1-\alpha} g\right)^{\frac{1}{\alpha-1}} = \bar{\kappa} = k_t^*, \\ c_t &= \frac{1}{P_t}\bar{\zeta} < \bar{\zeta} < \frac{\bar{\zeta}}{P_t^*} = c_t^*, \end{aligned}$$

□

Proposition. *3 Balanced Growth Path with $\tilde{Q} \in (0,\ 1)$ exists if and only if the solution to the following equations of Q_t and ϵ_t satisfies $0 < Q_t < 1$ and $\epsilon_t > 1$,*

$$(1-\gamma_F)\left(P_t^*\right)^{\eta} \epsilon_t^{\eta-1} c_t^* Q_t^{\theta_F(\eta-1)-\frac{\alpha}{1-\alpha}} - \frac{1-\gamma_H}{Q_t^{\theta_H(\eta-1)}}\left(P_t\right)^{\eta} \epsilon_t^{-\eta} c_t - Q_t g k_t = 0,$$

$$\frac{n+\delta+\rho+\frac{\alpha}{1-\alpha}\sigma g}{\epsilon_t - 1}\frac{1-Q_t}{Q_t} - \rho - \frac{\alpha}{1-\alpha}\sigma g + \frac{\alpha}{1-\alpha} g = 0,$$

where $k_t = \left(\frac{\delta+n+\rho+(\epsilon_t-1)Q_t g}{\alpha} + \frac{\sigma}{1-\alpha}g\right)^{\frac{1}{\alpha-1}} < \bar{\kappa}$.

Proof. According to the Hamiltonian, when $\epsilon_t > 1$ and $0 < Q_t < 1$, we get $\nu_t > 0$, meaning that the constraint of quality improvement is binding, $\hat{q}_t = g$. Equation (38), the Euler equation, implies that $k_t = \left(\frac{\delta+n+\rho+(\epsilon_t-1)Q_t g}{\alpha} + \frac{\sigma}{1-\alpha}g\right)^{\frac{1}{\alpha-1}} < \bar{\kappa}$.

Using an implication of the first-order conditions, such that $\frac{\lambda_t}{\nu_t} = \frac{\frac{1-Q_t}{Q_t} q_t}{(\epsilon_t - 1)K_t^i}$, we have

$$\begin{aligned} \phi_t &= \frac{\lambda_t}{\nu_t} R_t K_t^i + 2Q_t g - \rho + \hat{\nu}_t \\ &= \frac{\frac{1-Q_t}{Q_t} q_t}{\epsilon_t - 1}\frac{\alpha k_t^{\alpha-1}}{q_t} + 2Q_t g - \rho + \hat{\nu}_t \\ &= \frac{1-Q_t}{Q_t}\frac{\alpha k_t^{\alpha-1}}{\epsilon_t - 1} + 2Q_t g - \rho + \left[\hat{\lambda}_t + (\epsilon_t \hat{-} 1) + \hat{K}_t^i - \hat{q}_t - \left(\frac{1 \hat{-} Q_t}{Q_t}\right)\right] \end{aligned}$$

Since $\hat{C}_t^i = \hat{c}_t + \frac{\alpha}{1-\alpha}\hat{q}_t$, thus,

$$\phi_t = \frac{1-Q_t}{Q_t}\frac{\alpha k_t^{\alpha-1}}{\epsilon_t - 1} + 2Q_t g - \rho + \left[-\sigma \hat{c}_t - \frac{\alpha\sigma}{1-\alpha}\hat{q}_t - \hat{P}_t + (\epsilon_t \hat{-} 1) + \hat{k}_t + \frac{\alpha}{1-\alpha}\hat{q}_t - \hat{q}_t - \left(\frac{1 \hat{-} Q_t}{Q_t}\right)\right].$$

At steady state, we have $\phi_t = Q_t g$, $\hat{q}_t = \frac{\alpha}{1-\alpha} g$ and $\dot{c}_t = 0$. Using $\alpha k_t^{\alpha-1} = n + \delta +$

$\rho + \frac{\alpha}{1-\alpha}\sigma g + (\epsilon_t - 1)\phi_t$, we have

$$\begin{aligned}
\left(1 - \frac{1-Q_t}{Q_t}\right)\phi_t &= \frac{1-Q_t}{Q_t}\frac{n+\delta+\rho+\frac{\alpha}{1-\alpha}\sigma g}{\epsilon_t - 1} + 2Q_t g - \rho \\
&+ \left[-\sigma\hat{c}_t - \frac{\alpha\sigma}{1-\alpha}\hat{q}_t - \hat{P}_t + (\epsilon_t \hat{-} 1) + \hat{k}_t + \frac{\alpha}{1-\alpha}\hat{q}_t - \hat{q}_t - \left(\frac{1 \hat{-} Q_t}{Q_t}\right)\right] \\
2Qg - g &= 2Qg - g + \frac{1-Q_t}{Q_t}\frac{n+\delta+\rho+\frac{\alpha}{1-\alpha}\sigma g}{\epsilon_t - 1} - \rho - \frac{\alpha\sigma}{1-\alpha}\hat{q}_t + \frac{\alpha}{1-\alpha}\hat{q}_t
\end{aligned}$$

$$\frac{n+\delta+\rho+\frac{\alpha}{1-\alpha}\sigma g}{\epsilon_t - 1}\frac{1-Q_t}{Q_t} - \rho - \frac{\alpha\sigma}{1-\alpha}g + \frac{\alpha}{1-\alpha}g = 0$$

$$Q_t(\epsilon_t) = \frac{n+\delta+\rho+\frac{\alpha}{1-\alpha}\sigma g}{(\epsilon_t - 1)(\rho + \frac{\alpha}{1-\alpha}\sigma g - \frac{\alpha}{1-\alpha}g) + n + \delta + \rho + \frac{\alpha}{1-\alpha}\sigma g}.$$

□

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